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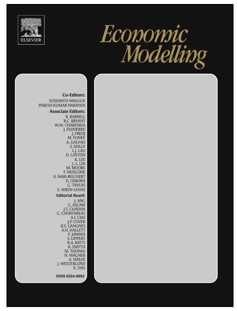

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The Transmission of Financial Shocks in an Estimated DSGE Model with Housing and Banking *

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Abstract

We develop a dynamic stochastic general equilibrium (DSGE) model with housing and banking to study the transmission of financial shocks between the financial and real sectors. A deterioration in the bank's balance sheet induced by financial shocks could have amplified and persistent impacts on real activities. The amplification of the shocks are originated from financial frictions tied to households and banks. We find that a disruption in bank net worth initiated by capital quality shocks generates a decline in household loans, house prices and output. Bank liquidity shocks also have negative effects on these variables. Housing preference shocks could generate a positive comovement between house prices and output. All these findings are qualitatively consistent with empirical evidence, suggesting that these financial shocks are critical to the dynamics of house prices and other macroeconomic variables.

JEL Classification: E32, E44, G01, G21, R31

Keywords: Banking; Housing; Business Cycle; DSGE Models; Financial Shocks; Financial Intermediation.

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1 Introduction

The events of the Great Recession of 2007-2009 have revealed that the dynamics of housing investment and house prices are affected by financial shocks via two channels: the household's balance sheet channel and the bank's balance sheet channel. On the one hand, a disruption in banks' net worth initiated by a financial shock weakens the ability of the banks in credit provisions, leading to a decline in house prices and housing investment. On the other hand, a disruption in households' net worth (i.e. housing values) induced by the shock attenuates the borrowing capacities of indebted households, and put more downward pressure on the housing market. The effects on the balance sheets of households and banks reinforce and interact each other, and hence produce a persistent and amplified impact on the housing market and the macroeconomy. These channels were arguably central to the turmoil during the Great Recession. Yet, a large class of the existing business cycle models ignore either the household's balance sheet channel or the bank balance sheet channel, while exploring the behaviors of the housing and financial markets in response to financial shocks.

In retrospect, the interaction between the two channels are far from being fully understood. For instance, what is the mechanism by which the two channels mentioned above interact in response to different financial disturbances? To what extent do financial factors affect the housing market and the macroeconomy as a whole? Why are both of channels critical to the housing and financial cycles? To address these questions, we extend Iacoviello and Neri (2010) to introduce financial intermediaries along the lines of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In doing so, it allows us to investigate the interaction between the balance sheet channels of households and banks. More importantly, the model is able to capture the joint behaviors between housing and financial variables observed in the data, at least in a qualitative sense.

In this paper, we find that a disruption in the bank's balance sheet initiated by a capital quality shock generates a decline in bank net worth, household loans, house prices and GDP. Credit squeezes induced by a bank liquidity shock also negatively affect household loans, house prices and GDP. Moreover, household loans, house prices and GDP move in the same direction in response to asset price shocks and productivity shocks. All these findings are consistent with empirical evidence. Last, the estimated model is successful in its ability in capturing the novel amplification mechanisms by which financial shocks affect the real economy through the household's balance sheet channel and the bank's balance sheet channel.

This paper is different from the existing literature in two dimensions. First, we combine the two sets of financial frictions in the model, which allows us to investigate the interaction between households and financial intermediaries through the two balance sheet channels–the household's balance sheet channel and the bank's balance sheet channel. The financial frictions tied to households and financial intermediaries in our model are not new, and they are originated from the two strands of literature. In particular, credit-constrained households are subject to financial frictions (collateral constraints), which are commonly used in the housing literature, such as Iacoviello (2005), Monacelli (2009), Iacoviello and Neri (2010), Mora-Sanguinetti and Rubio (2014) and Lee and Song (2015). With collateral constraints, a decline in housing values initiated by an exogenous shock could generate a pronounced effect on the behaviors of credit-constrained households through their balance sheet channels, and subsequently impinges on the real economy. Financial intermediaries face financial frictions (incentive constraints) while accumulating assets. The financial frictions tied to banks are widely employed in the recent financial literature, such as Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012), and Gertler and Kiyotaki (2015). With incentive constraints, a disruption in the bank's balance sheet could mitigate the bank's ability in lending funds to households, and therefore influences the dynamics of the housing market. The two financial frictions interact and reinforce each other over time, inducing a persistent and magnified effect on the real economy. However, none of these literature deal with both of financial frictions in a general equilibrium context, and therefore fail to account for the joint behaviors between the housing and financial markets.

Second, in the model we consider three types of financial shocks–capital quality shocks, bank liquidity shocks and asset price shocks. Although some of these shocks are familiar in the literature, many studies ignore the transmission of these shocks from the financial market to the housing market. Recent studies like Iacoviello (2015), Ferrante (2015), and Liu and Molise (2019), perhaps, are the most closed ones to our study, since financial frictions tied to households and banks are both considered in their works. But, they focus on different financial shocks. Aside from modeling differences, Iacoviello (2015) estimates a DSGE model where the recession is initiated by a repayment shock (redistribution shock), and find that losses sustained by banks can produce magnified and persistent effects on the business cycles. In absence of default penalties on borrowers (e.g. default interest and foreclosure), a negative repayment shock generates a positive wealth effect on the borrowers, and a negative wealth effect on the lenders. But, this is not a case in general. As observed in the Great Recession, a repayment shock had a negative effect on both lenders and subprimers, implying that wealth redistribution between them had not been materialized. In this regard, the responses of the borrowers to repayment shocks might be misleading when default penalties are mute in the model. Liu and Molise (2019) are subject to the same problem as Iacoviello (2015) as repayment shocks are introduced without default penalties. To address the drawbacks of these studies, we introduce the capital quality shock as a simple way to serve as an exogenous source of variations in bank net worth. Capital quality shocks are critical to the business cycles since they capture some form of economic obsolescence induced by the changes in technology and environmental standards.[1] The main discrepancy between the repayment shock and the capital quality shock is that the former produces a redistribution effect so that wealth are transferred from savers to borrowers in the event of default, whereas the latter does not. Ferrante (2015) extends Gertler and Karadi (2011) by including an endogenous mortgage default to study the effects of the mortgage-backed-securities (MBS) shocks and housing risk shocks on the real economy. The MBS shocks are motivated to capture the disturbance to the default rate on mortgages and their perceived risks, while the housing risk shocks are used to motivate an exogenous deterioration in the collateral value of mortgages for banks. Since our model does not focus on the role of mortgage default, the transmission of these financial shocks are beyond the scope of this study.

Financial distress can be attributed to either liquidity problems or solvency problems facing financial intermediaries. Any exogenous force that undermines the liquidity of banks will bring in a reduction in loans, and therefore house prices and GDP. To motivate the disturbance of bank liquidity, we introduce an exogenous shock to bank margins in a simple way to trigger the variations in the tightness of financial frictions tied to banks. Many existing studies, like Kiyotaki and Moore (2012), Jermann and Quadrini (2012), Del Negro, Eggertsson, and Ferrero and Kiyotaki (2017),

[1]For instance, a large fraction of capital might become obsolete due to technological advances and a tightening of environmental standards. As a result, the value of capital will fall, leading to a disruption in the bank's balance sheet. This occurs almost everywhere around the world, especially in the developing countries like China.

use this way to motivate a disruption in the financial market.[2] To our knowledge, we were the first to study the transmission of bank liquidity shocks in a general equilibrium model with housing and banking.

In addition to the financial shocks above, asset price shocks are also taken into account. In the model, a housing preference shock serves as an exogenous disturbance to the dynamics of house prices. Although this shock is not new, a large class of recent studies do not pay a closer attention to the joint behaviors between housing and financial quantities, such as the comovements of house prices with bank net worth, bank leverage and loans. We revisit the transmission of housing preference shocks in order to provide a better understanding of the interaction between the financial and housing cycles.

In general, financial shocks may differ one to another, in terms of the transmission channels. Because we conduct our research on the joint behaviors of the housing and financial markets from a different lens, the model developed in this paper can not only be used as a complement to its antecedents, but also provides an alternative framework to the family of business cycle models with housing and banking.

In what follows, section 2 presents some VAR evidence on bank net worth, household loans, house prices and GDP. Section 3 develops the baseline model. Section 4 characterizes the equilibrium of the model. Section 5 gives details in data description and parameter calibration. Section 6 estimates the structural and shock parameters of the model. In section 7, we examine the model's ability in fitting the VAR evidence, and discuss the transmission mechanism of financial shocks being considered. Section 8 concludes. All proofs and extended derivations are given in the appendix.

[2]Other studies use a different way to motivate a tightening of margins by introducing adverse selection, idiosyncratic risks and precautionary asset holdings to their frameworks. These studies, for example, include Williamson (1987), Aiyagari and Gertler (1999), Eisfeldt (2004), Curdia (2007), Kurlat (2013), Christiano, Motto and Rostagno (2009), Fostel and Geanakoplos (2008), Mendoza (2010), and Brunnermeier and Sannikov (2014).

2 A Structural VAR

Financial and housing cycles are likely affected by both financial and nonfinancial factors. In this paper, we seek to investigate the joint behaviors of the financial and housing variables in response to a particular financial shock. Specifically, a disruption in bank net worth and liquidity initiated by exogenous financial disturbances could tighten the bank's borrowing/lending constraint, inducing a decline in loans, house prices and output.

Given the theory we proposed in the model, we construct a structural VAR with real net worth (N), real household loans (B), real house prices (q), real GDP (GDP) from $1973Q1$ to $2011Q4$.[3] We adopt a Cholesky decomposition with the ordering indicated above, and assume that the structural shocks are orthogonal. The ordering is consistent with the transmission channel found in our theoretical model. Note that the VAR model is constructed with the short-run restrictions. In particular, an exogenous shock to net worth has a contemporaneous effect on household loans, house prices and GDP, and an exogenous shock to household loans has a contemporaneous effect on house prices and GDP with no contemporaneous effect on net worth. An exogenous shock to house prices has a contemporaneous effect on GDP, and do not affect net worth and household loans contemporaneously.

Figure 1 depicts impulse responses with the associated 95% bootstrapped confidence bands from the structural VAR. We document some key relationships among the variables of interest from the SVAR. Then we use an estimated DSGE model to match the VAR ecidence in three dimensions: (1) a negative response of household loans, house prices and GDP to a negative disruption in net worth (Figure 1, first row); (2) a negative response of house prices and GDP to a liquidity shock (Figure 1, second row) and; (3) a positive comovement of house prices and GDP in response to house price shocks (Figure 1, third row) and to output shocks (Figure 1, fourth row).

[3]The VAR includes a time trend, a constant and two lags of each series. All series are logged, deflated with the GDP deflator, and detrended with the HP-filter.

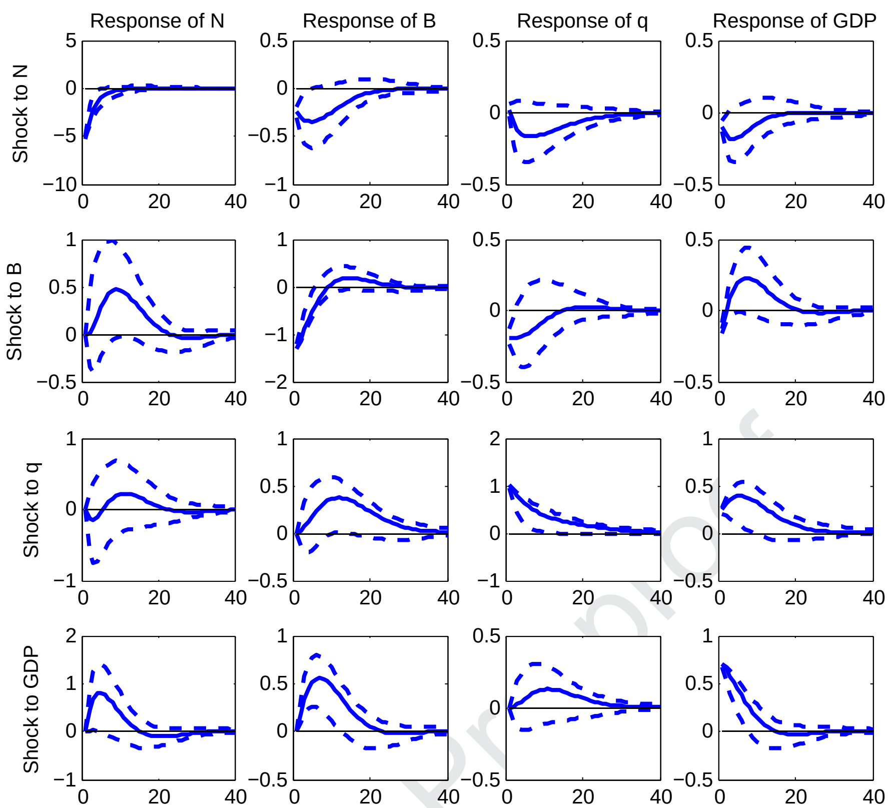

Figure 1: Impulse Responses from the Structural VAR

3 The Model

The model features multiple production sectors, heterogeneity of households, a borrowing friction faced by borrowers, and a financial friction faced by intermediaries. Following Gertler and Kiyotaki (2010), we formulate the banking system in a way that reflects a financial constraint associated with the bank's net worth when a bank issues deposits and makes loans. Aside from the introduction of the banking system, the baseline model closely follows a modified version of Iacoviello and Neri (2010).

There are two groups of households in the economy and each group has a unit measure of households. One group consists of patient households (net savers), and the other consists of impatient households (net borrowers). The economic size of each group is measured by its wage share, which is constant due to a unit elasticity of substitution in production. Households do not hold physical capital directly. Rather, they work, consume final goods, buy houses and deposit funds into or borrow from banks. In the equilibrium that we describe below, patient households turn out to be net savers and lend funds to impatient households and non-financial firms through the banking system. Conversely, impatient households turn out to be net borrowers in equilibrium, and in general they borrow funds from the banking system against their collateral which is tied to their housing values.

We assume that both the final goods sector and the housing sector operate under perfect competition, and that they produce consumption/investment goods and houses respectively using two different technologies. Firms in the final goods sector hire labor from households, and borrow funds from a bank to purchase intermediate physical capital. For simplicity, it is assumed that the final goods sector faces no further borrowing constraint and can commit to repay its debt obligations with its future gross profits to the creditor bank. In particular, a final goods producer obtains funds from a creditor bank by issuing state-contingent equities, and each unit of equity is a state-contingent claim to the future returns from one unit of new capital investment. Firms in the housing sector also hire labor from households and rent land as an input from patient households in order to produce houses.

The capital producer purchases final goods to be used as inputs to produce new capital and is subject to an adjustment cost. Firms in the capital sector are assumed to be owned by patient households, and all profits are redistributed to patient households through a lump sum transfer.

Banks are assumed to operate in a national retail market only. At the beginning of each period, banks obtain deposits from patient households and make loans to impatient households (household loans) and non-housing sectors (commercial loans). I rule out borrowing and lending in a wholesale market since inter-bank activities are beyond the scope of this paper.[4] As we mentioned earlier, banks are subject to an incentive constraint (deposit/lending constraint). Specifically, each bank with a given portfolio is constrained in its ability to issue deposits to savers and to make loans to borrowers. The incentive constraint can be motivated by government regulatory concerns or by standard moral hazard issues. Fundamentally, both incentive constraints and collateral constraints coexist and interact in the equilibrium so that banks are credit constrained in how much they can

[4] A notable paper regarding the inter-bank borrowing/lending is by Gertler and Kiyotaki (2010). For the case where the interbank borrowing is frictionless, the implication of the model would be similar to the baseline model. Otherwise, the results may change since interbank rates will lie between deposit rates and loan rates. The addition of an imperfect wholesale financial market will make the model less tractable.

accept in the form of deposits from patient households, and impatient households are credit constrained in how much they can borrow from banks. These two frictions interact and reinforce each other to induce a credit crunch during a financial crisis, and thus amplify the resulting economic recession.

3.1 Households

3.1.1 Patient Households (Savers)

We formulate the heterogeneity of households in a way that allows each group of households to differ in their discount factors and labor supply parameters. In the economy, there is a unit measure of patient households indexed by p. A representative patient household maximizes its lifetime utility function given by

$$U_p = E_0 \sum_{t=0}^{\infty} \beta_p^t A_{p,t} \{ \Gamma_p \ln(c_{p,t} - \tau_p c_{p,t-1}) + j_t \ln h_{p,t} - \frac{1}{1+\eta_p} (l_{pc,t}^{1+\epsilon_p} + l_{ph,t}^{1+\epsilon_p})^{\frac{1+\eta_p}{1+\epsilon_p}} \}.$$

Here, c_p, h_p, l_{pc} and l_{ph} are consumption, housing, hours supplied to the final goods sector and hours supplied to the housing sector, respectively. The last term in the bracket is the labor disutility function where η and ϵ are parameters that capture some degree of sector specificity. The formulation of the labor disutility function follows Horvath (2000), and with some choices of parameters it allows for imperfect labor mobility across production sectors. That is, hours are less perfect substitutes if $\epsilon > 0$; otherwise, they are perfect substitutes (e.g. $\epsilon = 0$). The terms $A_{p,t}$ and j_t captures the shocks to intertemporal preference and to housing preference. The parameter β_p is denoted as the discount factor for patient households. We assume $\beta_p > \beta_i$ in order to ensure that both impatient households and banks will be credit constrained in a neighborhood of the steady state. The term τ_p captures the degree of habits in consumption for patient households. The scaling factor $\Gamma_p = (1 - \tau_p)/(1 - \beta_p \tau_p)$ ensures that the marginal utility of consumption for patient households is $1/c_p$ in the steady state.

Patient households supply labor to producers, consume final goods, accumulate houses, and deposit or borrow funds with banks. They do not hold physical capitals directly. Rather, they hold land and rent it to the housing sector. Banks are assumed to be owned by patient households. In each period, patient households can receive a lump sum transfer from banks. As we will discuss in the next subsection, banks divert funds only to patient households upon their exit. The representative patient household faces the following budget constraint,

$$c_{p,t}+q_t h_{p,t}+p_{x,t}x_t+d_t = w_{pc,t}l_{pc,t}+w_{ph,t}l_{ph,t}+q_t(1-\delta_h)h_{p,t-1}+(p_{x,t}+R_t^x)x_{t-1}+R_t^d d_{t-1}+\Pi_t.$$

At the beginning of each period, the patient household chooses consumption $c_{p,t}$, housing $h_{p,t}$, land x_t, deposits d_t (loans if d_t is negative), hours $l_{pc,t}$ and $l_{ph,t}$ to maximize his/her utility subject to this budget constraint. The parameter δ_h denotes the depreciation rate of housing. The terms q_t and $p_{x,t}$ denote house prices and land prices, respectively. The terms $w_{pc,t}$ and $w_{ph,t}$ are real wages from supplying labor hours to final good and housing sectors. Deposits, d_t, are set in real terms here, and will yield a riskless return of R_t^d from period $t-1$ to period t. In addition, land is rented to the housing sector at a price of R_t^x. Finally, Π_t is the net average transfer received by the patient household from banks upon their exit.

The patient household's optimality conditions for consumption/deposits, houses, land, and hours taking prices as given are given by:

$$\begin{aligned}
1 &= \beta_p E_t(\frac{u_{cp,t+1}}{u_{cp,t}}R_{t+1}^d) && (1)\\
q_t &= \frac{A_{p,t}j_t}{u_{cp,t}h_{p,t}}+\beta_p E_t[\frac{u_{cp,t+1}}{u_{cp,t}}(1-\delta_h)q_{t+1}] && (2)\\
p_{x,t} &= \beta_p E_t[\frac{u_{cp,t+1}}{u_{cp,t}}(p_{x,t+1}+R_{t+1}^x)] && (3)\\
w_{pc,t} &= \frac{A_{p,t}}{u_{cp,t}}(l_{pc,t}^{1+\epsilon_p}+l_{ph,t}^{1+\epsilon_p})^{\frac{\eta_p-\epsilon_p}{1+\epsilon_p}}l_{pc,t}^{\epsilon_p} && (4)\\
w_{ph,t} &= \frac{A_{p,t}}{u_{cp,t}}(l_{pc,t}^{1+\epsilon_p}+l_{ph,t}^{1+\epsilon_p})^{\frac{\eta_p-\epsilon_p}{1+\epsilon_p}}l_{ph,t}^{\epsilon_p}, && (5)
\end{aligned}$$

with

$$u_{cp,t}=\Gamma_p(\frac{A_{p,t}}{c_{p,t}-\tau_p c_{p,t-1}}-\frac{\beta_p\tau_p A_{p,t+1}}{c_{p,t+1}-\tau_p c_{p,t}}),$$

where $u_{cp,t}$ denotes the marginal utility of consumption for patient households at time t.

Impatient Households (Borrowers)

Impatient households are assumed to hold no land or physical capital. In addition, they do not own banks and production sectors.[5] Rather, they work, consume, and are allowed to borrow funds

[5]In the equilibrium, all production sectors earn zero profit so it does not matter for the results if one assume that impatient households also own production sectors. In addition, a lump-sum transfer between banks and households plays a trivial role in the

from banks up to a fraction of the value of their houses. A representative impatient household chooses consumption $c_{i,t}$, housing $h_{i,t}$, hours $l_{ic,t}$ and $l_{ih,t}$, and loans b_t(saving if b_t is negative) to maximize its expected utility:

$$U_i = E_0 \sum_{t=0}^{\infty} \beta_i^t A_{p,t} \{ \Gamma_i \ln(c_{i,t} - \tau_i c_{i,t-1}) + j_t \ln h_{i,t} - \frac{1}{1+\eta_i} (l_{ic,t}^{1+\epsilon_i} + l_{ih,t}^{1+\epsilon_i})^{\frac{1+\eta_i}{1+\epsilon_i}} \},$$

subject to the following budget and collateral constraints:

$$\begin{aligned} c_{i,t} + q_t h_{i,t} + R_t^b b_{t-1} &= w_{ic,t} l_{ic,t} + w_{ih,t} l_{ih,t} + q_t(1-\delta_h) h_{i,t-1} + b_t \\ b_t &\leq m_t E_t (\frac{q_{t+1} h_{i,t}}{R_{t+1}^b}), \end{aligned}$$

where $w_{ic,t}$ and $w_{ih,t}$ are real wage rates from supplying hours to final goods sector and housing sector, respectively; and R_t^b is the rate of return on loans/savings incurred at date $t-1$. The collateral constraint characterizes the limit on the impatient household's ability to borrow up to the discounted future value of their houses. The term m_t is the loan-to-value (LTV) ratio that measures the effective degree of liquidity of houses. The larger m_t is, the greater is the value of housing as collateral to the impatient household. It is worth noting that in the model the term, m_t, captures an exogenous shock to the LTV ratio. The term, β_i, denotes the discount factor to the impatient household. Recall that we have $\beta_p > \beta_i$ in the model. This is a necessary condition that ensures impatient households are credit constrained in the neighborhood of the steady state, together with other parameters calibrated in the model. The term τ_i captures the degree of habits in consumption for impatient households. The scaling factor $\Gamma_i = (1-\tau_i)/(1-\beta_i \tau_i)$ ensures that the marginal utility of consumption for impatient households is $1/c_i$ in the steady state.

The impatient household's first-order conditions with respect to consumption, houses, loans, and hours taking prices as given can be written as:

model, as in Gertler and Kiyotaki (2010). In this regard, we do not assume that impatient households own banks for the sake of tractability.

$$\frac{A_{p,t}j_t}{u_{ci,t}h_{i,t}} + \beta_i E_t[\frac{u_{ci,t+1}}{u_{ci,t}}(1-\delta_h)q_{t+1}] = q_t - \lambda_{i,t} m_t E_t(\frac{q_{t+1}}{u_{ci,t}R^b_{t+1}}) \quad (6)$$

$$1 = \beta_i E_t(\frac{u_{ci,t+1}}{u_{ci,t}}R^b_{t+1}) + \frac{\lambda_{i,t}}{u_{ci,t}} \quad (7)$$

$$w_{ic,t} = \frac{A_{p,t}}{u_{ci,t}}(l^{1+\epsilon_i}_{ic,t} + l^{1+\epsilon_i}_{ih,t})^{\frac{\eta_i-\epsilon_i}{1+\epsilon_i}} l^{\epsilon_i}_{ic,t} \quad (8)$$

$$w_{ih,t} = \frac{A_{p,t}}{u_{ci,t}}(l^{1+\epsilon_i}_{ic,t} + l^{1+\epsilon_i}_{ih,t})^{\frac{\eta_i-\epsilon_i}{1+\epsilon_i}} l^{\epsilon_i}_{ih,t} \quad (9)$$

$$b_t = m_t E_t(\frac{q_{t+1}h_{i,t}}{R^b_{t+1}}), \quad (10)$$

with

$$u_{ci,t} = \Gamma_i(\frac{A_{p,t}}{c_{i,t} - \tau_i c_{i,t-1}} - \frac{\beta_i \tau_i A_{p,t+1}}{c_{i,t+1} - \tau_i c_{i,t}}),$$

where $u_{ip,t}$ denotes the marginal utility of consumption for impatient households at time t, and $\lambda_{i,t}$ denotes the Lagrange multiplier on the collateral constraint. The collateral constraint is binding, if the multiplier is positive.[6]

3.2 Banks

There are a large number of banks operating in a national financial market. Within the national financial market, we assume banks raise funds from patient households in a retail market rather than funds from inter-bank borrowing in a wholesale market. Because this paper primarily focuses on the interaction between the housing market and financial intermediation, the addition of inter-bank borrowing would go beyond the scope of this paper.

At the beginning of each period, a bank obtains deposits d_t from patient households, and pays a gross interest rate of R^d_{t+1} in the following period. In this economy, deposits are riskless one-period securities. At the same time, a bank makes household loans b_t to impatient households at a loan rate of R^b_{t+1}, and commercial loans to non-financial firms (e.g. final goods producers) in exchange for state-contingent equities from those firms at the price p_t. Household loans are subject to a friction because impatient households can only borrow up to a fraction of their housing values. For simplicity, we assume there are no frictions associated with commercial borrowing, since

[6]From equation (7), one may find that the collateral constraint is binding ($\lambda_i > 0$) in the steady state if and only if $R^d < \frac{1}{\beta_i}$. Given parameters calibrated in the model, we will observe later that this condition is satisfied.

banks are not only more efficient at evaluating and monitoring all activities of non-financial sectors than households, but are also more effective in enforcing contractual obligations. To motivate the logic above, we assume there are no costs for a bank performing these activities. Given these assumptions, a bank can issue frictionless loans to the final goods sector on the one hand, and a borrowing firm is able to offer the bank state-contingent equity on the other hand. In particular, each unit of equity is a state-contingent claim to the future returns from one unit of new capital investment.

Let s_t be the quantity of equities held by a representative bank, and n_t be the net worth of the bank in period t. Then the bank's net worth is equal to the gross payoffs from its assets (commercial loans and household loans) incurred at period $t-1$ net of deposit costs:

$$n_t = (Z_t + (1 - \delta_k)p_t)\psi_t s_{t-1} + R_t^b b_{t-1} - R_t^d d_{t-1}, \tag{11}$$

where the term δ_k is the capital depreciation rate. For simplicity, we assume the bank receives a unit of equity every time it issues commercial loans to non-financial firms to purchase an additional unit of capital. Accordingly, the quantity of equity in the economy remains the same as the quantity of capital.[7] The variable Z_t denotes the dividends paid in period t on the equities issued in period $t-1$. The variable ψ_t represents a capital quality shock, and follows an AR(1) process. As we will observe later, the market price of capital in the model is determined endogenously. Accordingly, one may think of this capital quality shock as an exogenous trigger to asset price dynamics. Note that the disturbance, ψ_t, is different from a standard rate of physical depreciation in that it can capture some forms of economic obsolescence.[8] In the model, this capital quality shock initially induces a deterioration in the banks' balance sheet. When the losses on the balance sheet induced by the shock cannot be fully absorbed by banks, a credit crunch may arise for the whole economy.

In period t, the flow-of-funds constraint for a bank can be written as,

$$p_t s_t + b_t = n_t + d_t. \tag{12}$$

The equation above implies that the bank's assets (loans) must equal its net worth plus liabilities (deposits).

[7]This assumption is widely used by papers like Kiyotaki, Michaelides and Nikolov (2011) and Kiyotaki and Gertler (2010). In the model, equity and capital exhibit a one-to-one relation in order to maintain tractability of the model. Thus, the price of equity equals to the price of capital in the model by this assumption.

[8]This shock might be thought of as capturing a disturbance to agents' anticipations over the quality/utilization of capital.

In the absence of some motive for paying dividends to households, banks may find it optimal to accumulate assets to the point where the financial constraint they face is no longer binding. In order to limit banks' ability to save to overcome financial constraint, we introduce an exogenous shock that induces the bank to leave the economy. In particular, a bank exits the economy. Upon exiting, a bank transfers all retained earnings to patient households.[9] At the same time, a new bank may, with probability $1-\sigma$, be established, leaving the number of banks constant in each period. In particular, a new bank takes over the business of an exiting bank and in the process inherits the skills of the exiting bank at no costs. The new bank receives a small fraction of the total assets of an ongoing bank as a "startup" fund from patient households. Recall that in each period a representative patient household receives an average net transfer Π_t from banks. The net transfer then must equal the funds transferred from exiting banks minus funds transferred to start-ups.

As we mentioned earlier, an endogenous financial constraint (incentive constraint) is introduced into the model. To motivate the endogenous financial constraint on the bank's ability to obtain funds in the retail financial market, we assume that the bank may divert a fraction θ_t of its assets to its owners (patient households) after it obtains funds (deposits) from the retail financial market. The bank's assets comprise the total value of equities held by the bank, $p_t s_t$, and consumer loans to impatient households, b_t. If the bank diverts its assets to its owners, it defaults on its debts (deposits) and is then forced to shut down. The creditors may reclaim the remaining fraction $1-\theta_t$ of funds. Given the risk of banks' default on their debts, creditors restrict the amount they lend to the bank at the beginning of each period. Accordingly, banks are constrained in their ability to obtain funds in the retail financial market, and in this way a financial constraint may arise. The bank's decision over whether to default on its debts must be made at the end of each period after the realization of the aggregate shock.

Let $V_t(s_t, b_t, d_t)$ be the value function of a bank at the end of period t, given its portfolio holdings (s_t, b_t, d_t). A bank will not default or divert funds, if it satisfies the incentive constraint given by,

$$V_t(s_t, b_t, d_t) \geq \theta_t(p_t s_t + b_t). \tag{13}$$

Unlike Gertler and Kiyotaki (2010), the term θ_t, is not a fixed parameter but an exogenous force

[9] It is important that the survival time is finite in the model so that patient households would finally get paid with dividends from banks even though the financial constraints are still binding.

that triggers the variations in bank liquidity. An increase in θ_t tightens the incentive constraint, and hence, the bank will be more likely to default on its obligations. Due to the increased risks of default, it is rational for depositors to restrict the amount they lend. In this way, a liquidity problem may arise. In such a scenario, banks are less willing to issue loans to their borrowers for any given level of net worth. We use this mechanism to motivate a disruption in credit provisions initiated by a liquidity shock. Let $\Lambda_{t,t+i}$ be the stochastic discount factor, which is equal to the patient households' marginal rate of substitution between consumption in period $t+i$ and that in period t. Since in the model banks are owned by patient households, they act on their behalf. In this regard, banks are assumed to have the same discount factor as patient households, $\beta_b = \beta_p > \beta_i$. In each period t, a representative bank maximizes the present value of its expected future net worth,

$$V_t(s_t, b_t, d_t) = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}, \tag{14}$$

subject to the incentive constraint and the flow-of-funds constraint given above. Recall that banks only pay dividends when they exit. Thus, the probability that a bank exits and pays the dividends after i periods from date t is $(1-\sigma)\sigma^{i-1}$ for $i \in [1, \infty)$. Given the bank's problem above, one can easily write the Bellman equation as follows,

$$V_{t-1}(s_{t-1}, b_{t-1}, d_{t-1}) = E_{t-1}\Lambda_{t-1,t}\{(1-\sigma)n_t + \sigma \max_{s_t, b_t, d_t} V_t(s_t, b_t, d_t)\}. \tag{15}$$

To solve the bank's problem, we first guess that the value function, V_t, is a time-varying linear function of (s_t, b_t, d_t) given by,

$$V_t(s_t, b_t, d_t) = \nu_{s,t} s_t + \nu_{b,t} b_t - \nu_{d,t} d_t \tag{16}$$

where $\nu_{s,t}$ is the marginal value of equities at the end of period t; $\nu_{b,t}$ is the marginal value of household loans; and $\nu_{d,t}$ is the marginal cost of deposits. In particular, these coefficients do not depend on the choices of individual banks.[10]

Let λ_t^b be the Lagrangian multiplier associated with the bank's incentive constraint. Given the conjectured form of the value function, one may use the Bellman equation together with the bank's incentive constraint and the flow-of-funds constraint to solve the bank's problem.[11] The objective

[10] One may treat these coefficients like prices. They are as being taken as given by individual banks.

[11] The nature of the maximization problem reflects the timing assumption that choices are made after the resolution of uncertainty.

function of the bank may be expressed as a Lagrangian,

$$£ = \nu_{s,t}s_t + \nu_{b,t}(n_t + d_t - p_t s_t) - \nu_{d,t}d_t + \lambda_t^b[(\nu_{b,t} - \theta_t)n_t - (\nu_{b,t} - \frac{\nu_{s,t}}{p_t})p_t s_t - (\theta_t - (\nu_{b,t} - \nu_{d,t}))d_t].$$

For the coefficients of the value function to be consistent with finite, positive choices of s, b and d in equilibrium, they must satisfy

$$\frac{\nu_{s,t}}{p_t} - \nu_{b,t} = 0 \tag{17}$$

$$(1 + \lambda_t^b)(\nu_{b,t} - \nu_{d,t}) = \theta_t \lambda_t^b \tag{18}$$

$$(\theta_t - (\nu_{b,t} - \nu_{d,t}))b_t + (\theta_t - (\frac{\nu_{s,t}}{p_t} - \nu_{d,t}))p_t s_t \leq \nu_{d,t}n_t. \tag{19}$$

Equation (17) indicates that the marginal value of equities must be equal to the marginal value of household loans, leading to no arbitrage opportunities across assets. That is, banks are indifferent between making commercial loans to non-financial sectors and making household loans to impatient households. Equation (18) implies that the marginal value of household loans exceeds the marginal cost of deposits if and only if the incentive constraint is binding ($\lambda_t^b > 0$). Accordingly, given equation (17) and equation (18), we will see later that in the model with financial frictions ($\lambda_t^b > 0$) there are excess returns on assets over deposits. Equation (19) is the bank's incentive constraint, and it states that the value of the bank's net worth must be at least as large as a weighted average value of its assets. When equation (19) holds with equality, financial frictions may arise.[12]

Due to the linearity of the problem, the conditions (17), (18) and (19) must hold in order for the equilibrium in the asset markets to be interior. Otherwise, banks would choose corner solutions. If corner solutions were chosen, other equilibrium conditions would be violated.[13]

In the next section, we proceed to characterize the model for two cases: with financial frictions ($\lambda_t^b > 0$) and without financial frictions ($\lambda_t^b = 0$). Furthermore, we will subsequently compare the models, and investigate how the financial friction amplifies the effect of an exogenous shock on the banks' balance sheet, and propagates this to the housing market and the economy as a whole.

[12]The complementary slackness condition, $[\nu_{d,t}n_t - ((\theta_t - (\nu_{b,t} - \nu_{d,t}))b_t + (\theta_t - (\frac{\nu_{s,t}}{p_t} - \nu_{d,t}))p_t s_t)]\lambda_t^b = 0$, implies that the incentive constraint (19) binds if $\lambda_t^b > 0$; otherwise, it may not bind.

[13]Banks choose $d = 0$ if and only if $\nu_b < \nu_d$, in which case it would violate the equilibrium condition $\nu_b \geq \nu_d$ for $\lambda^b \geq 0$; if banks choose $b = 0$, then the collateral constraint (10) implies either $q = 0$ or $h_i = 0$, in which case it would violate either the FOCs for housing or the necessary condition $h_i > 0$; if banks choose $s = 0$, then $k = 0$ and $c = 0$, in which case it would violate the necessary condition $c > 0$. In this regard, banks would not choose corner solutions when all equilibrium conditions hold.

3.2.1 Case 1: The Banking System with Financial Frictions

With financial frictions, banks are constrained in their ability to make loans to impatient households and non-financial firms. Given that the bank's incentive constraint is binding, equation (17) requires that the marginal value of equities relative to goods must equal the marginal value of household loans,

$$\frac{\nu_{s,t}}{p_t} = \nu_{b,t}. \tag{20}$$

In addition, equation (18) implies that the marginal value of household loans exceeds the marginal cost of deposits,

$$\nu_{b,t} - \nu_{d,t} > 0. \tag{21}$$

Combining equation (20) and equation (21), we obtain

$$\mu_t = \frac{\nu_{s,t}}{p_t} - \nu_{d,t} > 0, \tag{22}$$

where the term μ_t denotes the excess value of returns on assets over deposits.
Finally, given that banks are constrained by their funds to lend, one may rewrite equation (19) as,

$$p_t s_t + b_t = \phi_t n_t \tag{23}$$

with

$$\phi_t = \frac{\nu_{d,t}}{\theta_t - \mu_t}. \tag{24}$$

where the time varying parameter, ϕ_t, represents banks' leverage ratio. Note that the tightness of the incentive constraint depends positively on the fraction of assets that banks can divert, θ_t, and negatively on the excess value of assets over deposits, μ_t. Intuitively, the greater the fraction of assets that a bank can divert, the more likely it is that the bank will default on its deposits. Moreover, the greater the dispersion of returns between assets and liabilities, the less likely it is that a bank will default.

Let Ω_{t+1} be the marginal value of net worth at period $t+1$. Given the Bellman equation and the conjectured value function V_t, we can derive all undetermined coefficients of the conjectured value function, which are given as follows,

$$\nu_{b,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R^b_{t+1} \tag{25}$$

$$\nu_{d,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R^d_{t+1} \tag{26}$$

$$\nu_{s,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} \psi_{t+1} (Z_{t+1} + (1-\delta_k) p_{t+1}), \tag{27}$$

with

$$\Omega_{t+1} = 1-\sigma+\sigma(\nu_{d,t+1}+\phi_{t+1}\mu_{t+1}) \tag{28}$$

$$\mu_t = E_t\Lambda_{t,t+1}\Omega_{t+1}(R^k_{t+1}-R^d_{t+1}). \tag{29}$$

$$R^k_{t+1} = \psi_{t+1}\frac{Z_{t+1}+(1-\delta_k)p_{t+1}}{p_t}. \tag{30}$$

Appendix B gives full details in the determination of the coefficients of the conjectured value function, and verifies that the conjectured value function is linear in (s_t, b_t, d_t).

Proposition 1. *The value function $V_t(s_t, b_t, d_t)$ is linear in (s_t, b_t, d_t) if and only if equation (25) to equation (30) are satisfied.*

One may observe from equations (25) to (27), marginal values of assets and liabilities are equal to an augmented stochastic discount factor $\Lambda_{t,t+1}\Omega_{t+1}$ multiplied by their corresponding returns. Equation (28) states that the marginal value of net worth is a weighted average of the marginal values for exiting banks and ongoing banks. If an ongoing bank has an additional unit of net worth at date $t+1$, not only can it save the cost of deposits $\nu_{d,t+1}$, but it can also get additional benefits from accumulating its assets by a factor equal to the leverage ratio ϕ_{t+1}. Equation (29) implies that the excess value of assets relative to deposits is equal to an augmented stochastic discount factor multiplied by the spread of returns between assets and liabilities. Lastly, equation (30) is an expression for the gross rate of returns on equities. If a bank holds an additional unit of equity at a price of p_t today, it will receive a dividend Z_{t+1} and the value of the equity at a price p_{t+1} tomorrow after depreciation. Here, the term ψ_{t+1} captures a persistent shock that takes place in the next period.

Given equations (20) and (22), one can easily derive a relationship between assets and liabilities in terms of their returns, which is given as

$$R^k_{t+1} = R^b_{t+1} > R^d_{t+1}. \tag{31}$$

During a financial crisis, the excess return on assets over liabilities (the spread) will increase. To reduce the spread, banks must deleverage or accumulate more assets relative to debts. However, this deleveraging process takes a long time. So long as the spread is above its trend, financial factors act as a drag on the whole economy. Hence, the deleveraging process will slow down the recovery of the economy.

In the exposition, lower-case letters represent individual decision variables, and upper-case letters represent aggregate variables. Since all banks are homogenous in the model, given equation (23) we can obtain an expression for banks' aggregate assets given by

$$p_t S_t + B_t = \phi_t N_t, \tag{32}$$

where the variable S_t denotes the aggregate equity holdings at date t; B_t denotes the aggregate household loans; and N_t denotes the aggregate net worth of all banks. From now on, we drop all low-case letters and replace with upper-case letters to formulate the model.

Let $N_{o,t}$ be the aggregate net worth of ongoing banks at date t, and $N_{y,t}$ be the aggregate net worth of new banks. Then the aggregate net worth of all banks can be written as,

$$N_t = N_{o,t} + N_{y,t}. \tag{33}$$

Recall that banks may, with probability σ, survive to the next period. Thus, the aggregate net worth of ongoing banks must equal the sum of gross repayments of loans net of aggregate debt obligations, multiplied by the survival rate,

$$N_{o,t} = \sigma\{(Z_t + (1-\delta_k)p_t)\psi_t S_{t-1} + R_t^b B_{t-1} - R_t^d D_{t-1}\}. \tag{34}$$

Moreover, at the end of period $t-1$ banks may, with probability $1-\sigma$, exit from the banking system, while new banks enter the market with a fraction of funds transferred by patient households. For simplicity, we assume that patient households transfer a fraction $\xi/(1-\sigma)$ of the aggregate value of assets held by ongoing banks. Hence, the aggregate net worth of new banks is given by

$$N_{y,t} = \xi\{(Z_t + (1-\delta_k)p_t)\psi_t S_{t-1} + R_t^b B_{t-1}\}. \tag{35}$$

Finally, given the flow-of-funds constraints faced by individual banks, we can write the aggregate flow-of-funds constraint for all banks as follows,

$$p_t S_t + B_t = N_t + D_t. \tag{36}$$

Equation (36) states that the value of aggregate assets equals the sum of aggregate net worth and liabilities.

Before we proceed to the model without financial frictions in the next section, it is worth highlighting the mechanism by which an exogenous financial shock deteriorates the balance sheet of banks. During a financial crisis, a negative financial shock, directly reduces the value of assets (e.g. equities) held by banks and in turn puts downward pressure on the bank's net worth. Since banks are leveraged, the magnitude of the effect on a bank's balance sheet depends on the leverage ratio. The larger is the leverage ratio, the greater is the impact of the financial shock on the bank's balance sheet. In addition, there will be a second round effect on the banks' balance sheet as their net worth worsens. A decline in a bank's net worth reduces its ability to borrow in the retail financial market, causing a fire sale of bank's assets that further depresses the value of assets. Hence, the bank's balance sheet is further deteriorated. When the real economy strongly relies on the flow of funds from banks, it will suffer a huge loss during the crisis. The Great Recession is a good illustration of this mechanism.

3.2.2 Case 2: The Banking System without Financial Frictions

If there are no financial frictions, banks now convert deposits to loans without any excess returns for any time. Hence, one may treat the financial sector as a veil in this case. This is equivalent to thinking of banks playing no role in the model, and patient households directly lending funds to impatient households and nonfinancial firms with no financial intermediation.

In this case, the incentive constraint is not binding ($\lambda_t^b = 0$), implying that banks are not constrained in how much they can lend to borrowers. Given equation (17) and equation (18), the marginal value of assets must equal the marginal cost of liabilities,

$$\frac{\nu_{s,t}}{p_t} = \nu_{b,t} = \nu_{d,t}. \tag{37}$$

Combining equation (37) with equations (25) to (27), we can derive a perfect arbitrage condition which keeps returns on assets and liabilities equal over time:

$$R_{t+1}^k = R_{t+1}^b = R_{t+1}^d. \tag{38}$$

As we discussed earlier, a financial crisis is associated with an increase in the excess returns on assets over liabilities. Since there is no excess return in this case, a negative exogenous shock does not generate an amplified effect on the real economy.

Because banks play no role in this case, it is equivalent to thinking of banks disappearing from the model so that $N_t = 0$.[14] Moreover, all savings of patient households are converted to loans so

[14] One may solve the model with $N_t \neq 0$. But the implication of the model would be quite similar.

that the aggregate flow-of-funds constraint, equation (36) is replaced by

$$p_t S_t + B_t = D_t.$$

It is straightforward to solve the model without financial frictions. Subsequently, we will compare the dynamic paths of the key variables of interest across two cases in order to investigate the role of financial frictions in this framework.

3.3 Nonfinancial Firms

3.3.1 The Final Goods Sector

At date t, each final goods producer hires labors $l_{pc,t}$ and $l_{ic,t}$ from patient and impatient households, and pays a real wage of $w_{pc,t}$ and $w_{ic,t}$ to them, respectively. As we mentioned earlier, final goods producers face no borrowing constraints. Instead, they borrow funds from banks by issuing new state-contingent equities at price p_t. In particular, each unit of equity is a state-contingent claim to the future returns from one unit of capital investment. Conditional on funds borrowed from banks, they purchase new capitals as intermediate inputs from capital producers. In addition, we assume that final goods producers combine labors and capitals to produce final goods under a CRS technology in a Cobb-Douglas fashion. Due to perfect competition, the final goods producers earn zero profits state-by-state.

A representative final goods producer chooses hours $(l_{pc,t}, l_{ic,t})$ and intermediate capital k_t to produce final goods Y_t. The producer solves the firm's cost minimization problem as follows:

$$\min w_{pc,t} l_{pc,t} + w_{ic,t} l_{ic,t} + Z_t k_t.$$

subject to the production function given by,

$$Y_t = (A_{c,t}(l_{pc,t}^{\alpha} l_{ic,t}^{1-\alpha}))^{1-\mu_c} k_t^{\mu_c}. \tag{39}$$

Here Z_t denotes the dividends per unit of equity paid to creditor banks, the parameter $A_{c,t}$ measures labor productivity in the final goods sector, the parameter α denotes the labor income share of patient households, reflecting labor complementarity across different labor skills among households, and μ_c is the income share of capital used in the production of final goods. Note that the capital stock k_t is predetermined in period $t-1$. The optimality conditions for the final goods producer's problem are given by

$$
\begin{aligned}
w_{pc,t} &= \alpha(1-\mu_c)\frac{Y_t}{l_{pc,t}} && (40)\\
w_{ic,t} &= (1-\alpha)(1-\mu_c)\frac{Y_t}{l_{ic,t}} && (41)\\
Z_t &= \mu_c\frac{Y_t}{k_t}. && (42)
\end{aligned}
$$

Equation (40) and equation (41) are conditional wage functions for the final goods sector. Equation (42) states that dividends per unit of equity are equal to gross profits per unit of capital.

3.3.2 The Housing Sector

Housing producers build new houses with a CRS technology under perfect competition, but using different intermediate inputs. They hire labor $(l_{ph,t}, l_{ih,t})$ from the two groups of households and rent land x_{t-1} from patient households to produce new houses $I_{h,t}$. Thus, a representative housing producer solves the firm's cost minimization problem as follows;

$$\min w_{ph,t}l_{ph,t} + w_{ih,t}l_{ih,t} + R_t^x x_{t-1}.$$

subject to the production function given by,

$$I_{h,t} = (A_{h,t}(l_{ph,t}^{\alpha}l_{ih,t}^{1-\alpha}))^{1-\mu_h}x_{t-1}^{\mu_h}. \tag{43}$$

Here, the parameter $A_{h,t}$ measures labor productivity in the housing sector; the term R_t^x denotes the rental rate of land from period $t-1$ to period t; the parameter μ_h is the income share of land used to produce new houses. The optimality conditions for a representative housing producer's problem is then given by

$$
\begin{aligned}
w_{ph,t} &= \alpha(1-\mu_h)\frac{q_tI_{h,t}}{l_{ph,t}} && (44)\\
w_{ih,t} &= (1-\alpha)(1-\mu_h)\frac{q_tI_{h,t}}{l_{ih,t}} && (45)\\
R_t^x &= \mu_h\frac{q_tI_{h,t}}{x_{t-1}}. && (46)
\end{aligned}
$$

3.3.3 The Capital Goods Sector

Capital goods producers produce new capital using final goods as inputs, and incur an adjustment cost. They sell capital to the final goods sector at price p_t. We assume that the adjustment cost function is convex in the net growth rate of capital investment and takes the following form:[15]

$$f(\frac{I_{k,t}}{I_{k,t-1}})I_{k,t} = (\frac{I_{k,t}}{I_{k,t-1}} - 1)^2 I_{k,t}. \tag{47}$$

A capital producer chooses capital investment $I_{k,t}$ to maximize the objective function:

$$\max E_t \sum_{i=t}^{\infty} \Lambda_{t,i} \{p_i I_{k,i} - [1 + (\frac{I_{k,i}}{I_{k,i-1}} - 1)^2] I_{k,i}\},$$

where $\Lambda_{t,i}$ is the patient household's stochastic discount factor from date i to date t.[16] The optimality condition then yields the price function for capital given by,

$$p_t = 1 + (\frac{I_{k,t}}{I_{k,t-1}} - 1)^2 + 2(\frac{I_{k,t}}{I_{k,t-1}} - 1)\frac{I_{k,t}}{I_{k,t-1}} - 2\Lambda_{t,t+1}(\frac{I_{k,t+1}}{I_{k,t}} - 1)(\frac{I_{k,t+1}}{I_{k,t}})^2. \tag{48}$$

Note that profits will arise only outside of the steady state, and will be redistributed to patient households by a lump sum transfer.

3.4 Shock Processes

We assume that all shocks to be estimated in the model follow the AR(1) process and evolve in a log-linear fashion:

$$\ln \psi_t = \rho_k \ln \psi_{t-1} + u_{k,t} \tag{49}$$

$$\ln A_{c,t} = \rho_c \ln A_{c,t-1} + u_{c,t} \tag{50}$$

$$\ln A_{h,t} = \rho_h \ln A_{h,t-1} + u_{h,t} \tag{51}$$

$$\ln A_{p,t} = \rho_p \ln A_{p,t-1} + u_{p,t} \tag{52}$$

$$\ln m_t = (1-\rho_m) \ln m + \rho_m \ln m_{t-1} + u_{m,t} \tag{53}$$

$$\ln j_t = (1-\rho_j) \ln j + \rho_j \ln j_{t-1} + u_{j,t} \tag{54}$$

$$\ln \theta_t = (1-\rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + u_{\theta,t} \tag{55}$$

[15]This cost function satisfies $f(1) = f'(1) = 0$ and $f''(\frac{I_{k,t}}{I_{k,t-1}}) > 0$. Therefore, the aggregate production function of capital goods producers exhibits a decreasing returns to scale in the short-run and a constant returns to scale in the long-run.

[16]Since patient households own capital goods firms, we assume that capital producers are as patient as patient households.

where ρ_k, ρ_c, ρ_h, ρ_p, ρ_m, ρ_j and ρ_θ are persistence parameters; $u_{k,t}$, $u_{c,t}$, $u_{h,t}$, $u_{p,t}$, $u_{m,t}$, $u_{j,t}$ and $u_{\theta,t}$ are innovations that are serially uncorrelated with zero mean and standard deviations σ_k, σ_c, σ_h, σ_p, σ_m, σ_j and σ_θ, respectively. The term m, j and θ are steady state values for LTV ratio, housing preference rate and the fraction of assets diverted, respectively.

These shock parameters later will be estimated, using a Bayesian approach as described in An and Schorfheide (2007).

4 Equilibrium

To close the model, we require market clearing for goods, houses, and equities/capital as follows,

$$Y_t = C_t + (1 + (\frac{I_{k,t}}{I_{k,t-1}} - 1)^2) I_{k,t} \tag{56}$$

$$I_{h,t} = H_t - (1 - \delta_h) H_{t-1} \tag{57}$$

$$S_t = I_{k,t} + (1 - \delta_k) K_t \tag{58}$$

with

$$C_t = c_{p,t} + c_{i,t} \tag{59}$$

$$H_t = h_{p,t} + h_{i,t} \tag{60}$$

$$K_{t+1} = \psi_{t+1}(I_{k,t} + (1 - \delta_k) K_t). \tag{61}$$

Note that equation (61) is the law of motion for capital in presence of an exogenous capital quality shock. Land per capita is fixed and normalized to one, that is $x_t = 1$.[17]

Definition 1. *A competitive equilibrium consists of a sequence of prices for* $t \in [1, +\infty]$

$$(p_t, q_t, p_{x,t}, R^d_{t+1}, R^b_{t+1}, R^k_{t+1}, R^x_{t+1}, w_{pc,t}, w_{ph,t}, w_{ic,t}, w_{ih,t})$$

and the sequence of shadow prices ($\nu_{d,t}, \nu_{b,t}, \nu_{s,t}, \lambda^i_t, \mu_t$) *such that the sequence of individual quantities*

$$(c_{p,t}, c_{i,t}, h_{p,t}, h_{i,t}, l_{pc,t}, l_{ph,t}, l_{ic,t}, l_{ih,t})$$

[17] In general, land is not fixed over time. But, land per capita is relatively fixed. One way to motivate the constant supply of land per capita is to assume that the growth rate of land equals that of population. For simplicity, we assume that there is no population growth in the economy so as to the supply of land per capita is fixed over time.

and the sequence of aggregate quantities

$$(Y_t, C_t, H_t, K_{t+1}, S_t, I_{h,t}, I_{k,t}, Z_t, N_t, B_t, D_t, \phi_t)$$

satisfy the following conditions:

1. *Given prices, households maximize their expected life-time utility functions, (1)-(10).*

2. *Given prices, final good producers minimize their costs, (40)-(42).*

3. *Given prices, housing producers minimize their costs, (44)-(46).*

4. *Given prices, capital goods producers maximize their profits, (48).*

5. *Given the conjectured value function V_t, banks maximize their present value of future net worth, (17)-(19).*

6. *The conjectured value function V_t is linear and must satisfy equations (25) to (30).*

7. *Banks are indifferent between making commercial loans and consumer loans, (31).*

8. *The banks' leverage ratio satisfies equations (32) and (33).*

9. *All markets clear and satisfy equations (56) to (61).*

10. *Shocks follow the AR(1) process given by equations (49)-(55).*

Then, given eleven prices $(p_t, q_t, p_{x,t}, R^d_{t+1}, R^b_{t+1}, R^k_{t+1}, R^x_{t+1}, w_{pc,t}, w_{ph,t}, w_{ic,t}, w_{ih,t})$ together with five shadow prices $(\nu_{d,t}, \nu_{b,t}, \nu_{s,t}, \lambda^i_t, \mu_t)$, eight individual quantities $(c_{p,t}, c_{i,t}, h_{p,t}, h_{i,t}, l_{pc,t}, l_{ph,t}, l_{ic,t}, l_{ih,t})$ and twelve aggregate quantities $(Y_t, C_t, H_t, K_{t+1}, S_t, I_{h,t}, I_{k,t}, Z_t, N_t, B_t, D_t, \phi_t)$ can be determined by a dynamic system of 36 equations. Please see full details of the dynamic system of the model in Appendix C.

5 Data and Calibration

In this section, we calibrate the parameters used in the baseline model to match the targeted ratios of key economic quantities that are observed in the data. These targeted ratios are estimated by using nine time series from $1973Q1$ to $2011Q4$: real private consumption expenditure, real private residential fixed investment, real private non-residential fixed investment, real house prices, real commercial loans, real household loans, real deposits, real net worth and nominal interest rates. Each time series being used in the model is HP filtered with $\lambda = 1,600$ and detrended. See Appendix A for data descriptions. The model period is quarterly.[18]In particular, depreciation rates for capital and housing, capital and land shares in the production, and the housing preference parameter are calibrated to match consumption, residential investment, nonresidential investment, the value of land and housing wealth to GDP ratios observed in the US data. In addition, we calibrate parameters used in the bank's problem (θ, ξ) in order to match an average credit spread and an average economy-wide leverage ratio observed from the data in all US commercial banks. Following Iacoviello and Neri (2010), we do not use the time series of the labor employment to calibrate the labor supply parameters since in any multi-sector model the link between value added of the sector and available measures of total hours worked in the same sector is tenuous. For this reason, we allow for measurement error in total hours in the consumption and housing sectors.

The discount factors β_p and β_i, depreciation rates for capital and houses δ_k and δ_h, capital and land shares in each production sector μ_c and μ_h, the housing preference weight in the utility function j, the fraction of funds transferred from patient households ξ, and the fraction of assets that can be diverted from banks to patient households θ are calibrated by the model itself. Table 1 summarizes our calibrations. All calibrated parameters together with the other parameters that are set explicitly yield the steady-state ratios that are consistent with the data. A summary of steady-state ratios of the baseline model is presented in the Table 2.

The discount factor for patient households β_p is set to 0.9925, implying that a steady-state annual real interest rate of 3%. I arbitrarily set the discount factor for impatient households to $\beta_i = 0.965$ since the value of β_i has little effect on the dynamics of the model. The only restriction on β_i is that its value must be lower than β_p to ensure that the borrowing constraint of impatient

[18]The model period used in existing literature varies from one to another. For instance, in Davis and Heathcote (2005) the model period is one year since they attempt to capture the length of time between starting to plan new investment and the resulting increase in the capital stock being in place. They find that business cycle statistics, however, are virtually same for both quarterly and annual period lengths. Others like Gomme et al. (2001) and Fisher (2007) set the time to built for residential investment to one quarter, and the time to built for non-residential investment to four quarters in order to account for the fact that nonresidential investment lags residential investment. Since the lead-lag pattern between residential investment and nonresidential investment is beyond our interest in this paper, the length of the time between starting to plan both residential and nonresidential investment and the resulting increase in their corresponding stocks being in place is then set to one quarter.

Table 1: Calibrated Parameters

Parameters	Description	Value
β_p	Discount factor for savers	0.9925
β_i	Discount factor for borrowers	0.965
δ_k	Depreciation rate of capital	0.0188
δ_h	Depreciation rate of houses	0.0106
μ_c	Share of capital	0.1795
μ_h	Share of land	0.2519
θ	Fraction of assets diverted by banks	0.4033
ξ	Fraction of funds transferred to new banks	0.0005
j	Housing preference weight	0.1332
m	Loan-to-value ratio	0.85
σ	Survival rate of banks	0.975

households is binding around the neighborhood of the steady state. In general, the lower is the value of β_i, the more likely will the borrowing constraint bind away from the steady state.

The survival rate of banks is arbitrarily set equal to 0.975, implying that banks survive for ten years on average.[19] Although the expected survival time of banks may be longer than ten years, we only require that the expected time horizon is finite so that dividends paid by banks are guaranteed while the financial constraints are still binding. An alternative choice of the survival rate would not have a substantive effect on the business cycle properties reproduced by the model. The fraction of funds transferred from patient households ξ, and the fraction of assets that can be diverted from banks to patient households θ are set equal to 0.0005 and 0.4033, respectively, in order to match two targets: an average leverage ratio of 6 from all US commercial banks and an average annual interest spread of 1%. According to the Table of Assets and Liabilities of All Commercial Banks in the United States from the Fed's, the average leverage ratio is 6 approximately.[20] We set the target for the annual interest spread to 1%, based on the pre-2007 spread as a rough average of

[19] The survival time $= \frac{1}{1-\sigma}$. Here, we choose the same survival rate as the one used by Gertler and Kiyotaki (2010).

[20] we use time series of Household Loans (Residential Real Estate Loans included), C&I loans (Nonresidential Real Estate Loans included) and Net Worth for all U.S. commercial banks from $1973:Q1$ to $2011:Q4$ to calculate the average leverage ratio, that is, $LeverageRatio = \frac{ConsumerLoans + C\&ILoans}{NetWorth}$.

Table 2: Steady-State Ratios

Variables	Interpretation	Value
$4 \times (R_d - 1)$	Annual real interest rate	3%
$4 \times (R_k - R_d)$	Annual interest spread	1%
C/GDP	Consumption to GDP ratio	83%
I_k/GDP	Nonresidential investment to GDP ratio	11%
qI_h/GDP	Residential investment to GDP ratio	6%
$K/(4 \times GDP)$	Nonresidential capital stock to annual GDP ratio	1.46
$qH/(4 \times GDP)$	Housing wealth to annual GDP ratio	1.41
$p_x/(4 \times GDP)$	Value of land to annual GDP ratio	0.50
ψ	Average leverage ratio	6

Note: Our definition of GDP is the sum of consumption, residential investment and nonresidential investment.

the following three spreads: BAA corporate bond rates versus government bonds, mortgage rates versus government bonds, and commercial paper rates versus T-Bill rates.

We choose the depreciation rates for capital $\delta_k = 0.0188$ and the capital share in the consumption goods sector $\mu_c = 0.1795$ together with other calibrated parameters to hit a steady-state nonresidential investment to GDP ratio of 11% and a steady-state nonresidential capital stock to annual GDP ratio of 1.46.[21] In addition, the depreciation rate for housing is set equal to $\delta_h = 0.0106$ and the land share in the housing production function is set equal to $\mu_h = 0.2519$, implying a steady-state residential investment to GDP ratio of 6% and a steady-state land value to annual GDP ratio of 0.50. The calibrated value of δ_h is consistent with the calibration from most of the housing literature such as Davis and Heathcote (2005) and Iacoviello and Neri (2010).[22] The calibrated value of μ_h seems to be also consistent with previous housing literature, see Davis and Palumbo (2008), Saiz (2010), Kiyotaki, Michaelides and Nikolov (2011), and Lloyd-Ellis, Head

[21] Note that the depreciation rate for capital $\delta_k = 0.0188$ in our model is slightly lower than that in Iacoviello and Neri (2010) and Iacoviello (2005) while the target of the nonresidential investment to GDP ratio used in this paper is lower than the one used by those studies.

[22] Greenwood and Hercowitz (1991) calibrate a larger depreciation rate of 0.078 for residential structures; but Davis and Heathcote (2005) calibrate a slightly smaller depreciation rate of 0.0157; and Iacoviello and Neri (2010) calibrate a depreciation rate of 0.01.

and Sun (2014).[23] The preference weight for housing in the utility function is set at $j = 0.1339$. Given the value of input shares in the housing production function and other parameters calibrated, the choice of housing preference weight implies a steady-state housing wealth to annual GDP ratio of 1.41.

According to the data from the Finance Board's Monthly Survey of Rates and Terms on Conventional Single-Family Non-farm Mortgages Loans, the average loan-to-value ratio for homebuyers is about 0.76 between 1973 and 2006. Since the LTV ratio in the model refers to the fraction of housing value that can be borrowed by those who are credit constrained (impatient households), the value of the ratio for those credit constrained homebuyers might be greater than the average value for overall homebuyers. In this regard, we choose a conservative LTV ratio equal to 0.85.

The labor income share of credit-constrained households, $1 - \alpha$, is difficult to estimate due to data availability. More importantly, different approaches and data sources generate different estimated values. Kiyotaki, Michaelides and Nikolov (2011) report that the fraction of constrained home owners in the population is 13.9%, and the fraction of tenants is 36% in the early $1990s$. Suppose we treat tenants as constrained households, then the fraction of constrained households is about 50%. Jappelli (1990) uses the 1983 Survey of Consumer Finances to estimate a fraction of 20% population that are credit constrained. Iacoviello (2005) gives a wage share of credit constrained households of 36%, based on a limited information approach. In the model, we set $1-\alpha = 0.36$, following Iacoviello (2005) since it is within the range of estimates reported by most of the existing literature.

6 Bayesian Estimation

We use a Bayesian approach as described in An and Schorfheide (2007) to estimate the parameters that are hard to calibrate. We estimate the model using U.S. quarterly data from $1973Q1$ to $2011Q4$. We choose the prior distributions of the parameters that are consistent with previous studies such as Iacoviello and Neri (2010) and Iacoviello (2015). Table 3 reports the prior distributions of the parameters estimated. In the estimation, we use as many shocks as observables. Though some shocks, like the intertemporal preference shock and the LTV shock, are not particu-

[23]Those literature calibrate a relative share of land in the price of housing within a range from 25% to 30%.

Table 3: Prior Distribution of the Parameters

Parameters	Distribution	Mean	Standard Deviation
τ_p	Beta	0.50	0.075
τ_i	Beta	0.50	0.075
η_p	Gamma	0.50	0.10
η_i	Gamma	0.50	0.10
ϵ_p	Normal	1.00	0.10
ϵ_i	Normal	1.00	0.10
ρ_c	Beta	0.80	0.10
ρ_h	Beta	0.80	0.10
ρ_k	Beta	0.80	0.10
ρ_j	Beta	0.80	0.10
ρ_p	Beta	0.80	0.10
ρ_m	Beta	0.80	0.10
ρ_θ	Beta	0.80	0.10
σ_c	Inverse Gamma	0.001	0.01
σ_h	Inverse Gamma	0.001	0.01
σ_k	Inverse Gamma	0.001	0.01
σ_j	Inverse Gamma	0.001	0.01
σ_p	Inverse Gamma	0.001	0.01
σ_m	Inverse Gamma	0.001	0.01
σ_θ	Inverse Gamma	0.001	0.01

Note: The upper panel reports the prior distribution of structural parameters, and the lower panel reports the prior distribution of shock parameters.

larly of our interest, these shocks may affect the relative importance of other shocks in explaining the dynamics of investments, house prices, loans and output. The observables we used are real private consumption, real residential investment, real nonresidential investment, real house prices, real household loans, real commercial loans and real net worth. Appendix A gives data description in details. In general, the choice of observables does matter for the posteriors estimated. The reason we select these observables in the estimation is that they are closely related to the variables of interest in the VAR, since our goal of this study is to use the estimated model to match the VAR impulse responses. Table 4 reports the posterior mean and 90% confidence interval for the parameters estimated.[24]

Table 4: Posterior Distribution of the Parameters

Parameters	Mean	5%	95%
τ_p	0.5181	0.4370	0.5957
τ_i	0.3577	0.2807	0.4273
η_p	0.5273	0.3800	0.6948
η_i	0.3991	0.2850	0.5138
ϵ_p	1.0792	0.9178	1.2317
ϵ_i	1.0721	0.9397	1.2045
ρ_c	0.9394	0.9060	0.9722
ρ_h	0.8785	0.8053	0.9393
ρ_k	0.6912	0.6548	0.7208
ρ_j	0.8867	0.8468	0.9271
ρ_p	0.8103	0.7431	0.8891
ρ_m	0.8105	0.7720	0.8429
ρ_θ	0.6145	0.5381	0.6819
σ_c	0.0092	0.0080	0.0105
σ_h	0.0267	0.0245	0.0290
σ_k	0.0115	0.0105	0.0127
σ_j	0.1766	0.1197	0.2302
σ_p	0.0293	0.0258	0.0326
σ_m	0.0362	0.0325	0.0399
σ_θ	0.0209	0.0180	0.0238

Note: The upper panel reports the posterior distribution of structural parameters, and the lower panel reports the posterior distribution of shock parameters.

Most shocks are quite persistent, with autocorrelation coefficients ranging from 0.81 to 0.94,

[24]The estimation is based on a sample of $10,000$ draws. Draws from the posterior distribution of the parameters are obtained using the random walk version of the Metropolis-Hastings (MH) algorithm. The scale used for the jumping distribution in the MH algorithm is set to 0.40, reflecting an acceptance rate of 35%, approximately, in the algorithm. The number of parallel chains for the MH algorithm is set to 1. Convergence was carefully assessed by using the "MCMC univariate diagnostics". We obtain relative stability and convergence in all measures of the parameter moments, implying that our posteriors are sensible. In addition, the posterior distributions are closed to normal, and the mode calculated from the numerical optimization of the posterior kernel seems not too far away from the mode of the posterior distribution, implying a strong confidence in our posteriors.

except for capital quality shocks and liquidity shocks. These twos are less persistent, relative to others, with autocorrelation coefficients of 0.6912 and 0.6145, respectively. The volatility of housing preference shock are found to be much higher than the other shocks, with a standard deviation of 0.18. Turning to the labor supply parameters, the labor supply elasticities of the two types of households are quite similar ($\eta_p = 1.0792$ and $\eta_i = 1.0721$), and both households exhibit little preference for mobility across sectors ($\epsilon_p = 0.5273$ and $\epsilon_i = 0.3991$). In contrast to Iacoviello and Neri (2010), the degree of habits in consumption is larger for patient households than for impatient households ($\tau_p = 0.5181$ and $\tau_i = 0.3577$). One possible reason for this may be that in our baseline model patient households do not hold capital directly, they cannot smooth consumption by accumulating capital. Another reason may be that savings (deposits) are more vulnerable to a disruption of the financial market, and patient households are less able to smooth consumption through saving, relative to that in the model without financial frictions. Accordingly, a larger degree of habits in consumption is needed for patient households in order to match the persistence of aggregate consumption in the data.

7 The Transmission of the Shocks

7.1 The Model Dynamics Versus Structural VAR Evidence

Figure 2 reports the impulse responses from the model with and without financial frictions. We compare them with the VAR impulse responses to assess the key properties of the model, and its consistency with empirical evidence.

The first row of Figure 2 shows the impulse responses to a negative capital quality shock. The capital quality shock immediately reduces bank net worth. It then tightens the bank's borrowing constraint, inducing a drop in loans issued. The effect of decline in net worth on loans is associated with the leverage ratio. The larger is the leverage, the greater is the impact on loans. The contraction of loans, in turn, puts a downward pressure on housing demand. Eventually, house prices fall. While both consumption and investment decline over time in response to the shock, GDP falls. In addition, we find that the model with financial frictions does well in capturing the VAR impulse responses to a negative shock to bank net worth. It is worth noting that the model without financial frictions performs poorly in matching empirical evidence. Both loans and house

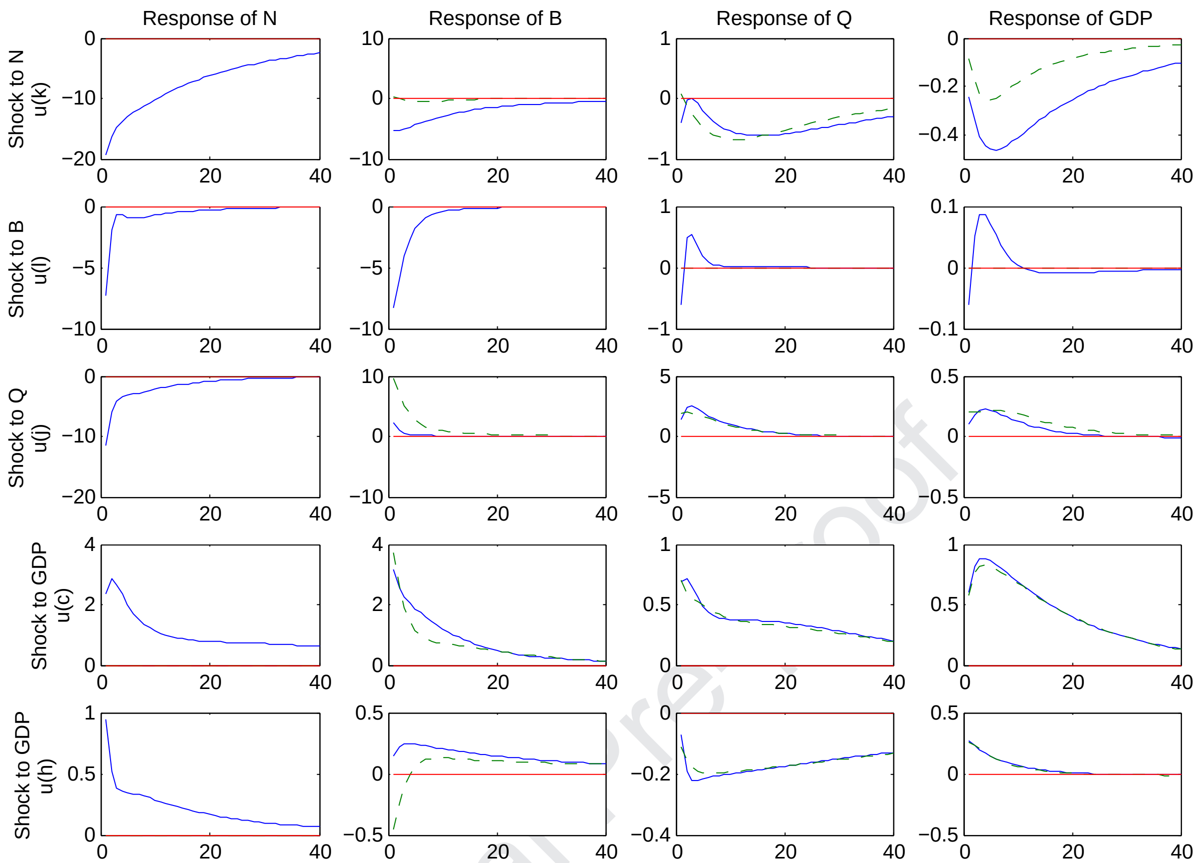

Figure 2: Impulse Responses of the Estimated Model

Note: the horizontal axis measures time horizon in quarters, and the vertical axis measures percent deviation from the steady state. The solid lines plot the impulse responses from the model with financial frictions, and the dash lines plot the impulse responses from the model without financial frictions. Each shock is one standard deviation in size.

prices rise initially, and the former does not vary much over time in response to the shock, in which they are not consistent with empirical evidence.

The second row of Figure 2 shows the impulse responses to a negative bank liquidity shock (equivalently, a negative lending shock). The baseline model performs well in capturing the negative responses of net worth, households loans, house prices and GDP to the shock, as observed in the data. In the case without financial frictions, a leverage shock has no effect on the dynamics of variables, as the incentive constraint is slack.

The third row of Figure 2 shows the impulse responses to a positive housing preference shock. The model produces a positive comovement of household loans, house prices and GDP, which is consistent with empirical evidence. This finding is also along the lines of Iacoviello (2015).

The fourth and fifth rows of Figure 2 show the impulse responses to productivity shocks in the nonhousing sector and the housing sector, respectively. The baseline model produces the positive responses of net worth, loans, house prices and GDP to a nonhousing productivity shock, as observed in the data. Given a housing productivity shock, the response of house prices is counterfactual. Not surprisingly, this is a general outcome of the supply-side shocks. In the case without financial frictions, it generates a counterfactual response of loans to a positive housing productivity shock.

7.2 More on the Model Dynamics

7.2.1 The Capital Quality Shocks

Figure 3 plots the impulse responses of the key variables of interest to a negative capital quality shock. The solid line represents the dynamic paths for the model with financial frictions, and the dash line represents the model without financial frictions.

With financial frictions, a decline in capital quality immediately leads to a decrease in bank net worth. This huge decline in net worth is fundamentally a product of a high bank leverage ratio and a decline in the market price of equities arising from the fire sale of the bank's assets. In particular, an exogenous shock affects the banks' net worth in two ways. First, an initial capital quality shock directly reduces the value of equities held by banks, and hence their net worth. Because the bank is highly leveraged, the effect on its net worth would be magnified by a factor equal to the bank's leverage ratio. Second, the decline in the net worth then tightens the bank's borrowing constraint, causing a fire sale of the bank's assets (equities). Eventually, the price of the equity falls. This second round effect further depresses the value of the bank's equity and thereby induces a huge loss in the bank's net worth. In addition, although deposits slightly decrease initially, they eventually fall over time due to the increase in the tightness of the bank's borrowing constraint.

In order to respond to the decline in the bank's net worth, the bank reduces the amount of funds lent to impatient households and final goods firms. This leads to a large decline in commercial

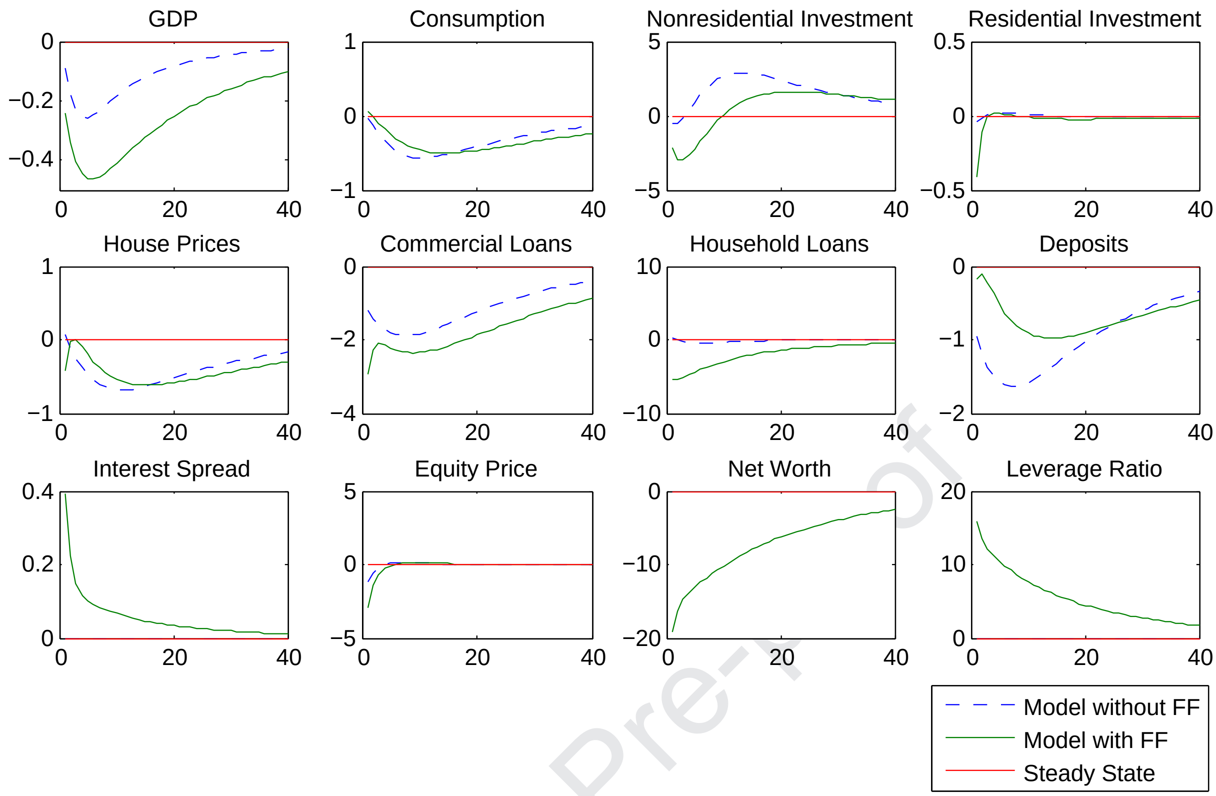

Figure 3: Impulse Responses to a Negative Capital Quality Shock

loans and household loans. Since the movement in commercial loans is ultimately responsible for the change in business investment, the latter falls as commercial loans decline. As a result, the capital stock and output fall.

The aggregate demand for housing or final goods depends on the movement of the individual demand in each group of households. For impatient households, individual demand for housing falls as the amount of household loans received from banks decreases. Individual demand for final goods, however, initially increases, reflecting the fact that impatient households substitute towards final goods and away from housing due to their low patience and the anticipated depreciation of house prices. Individual demand for final goods eventually falls as their budget constraints get tighter. For patient households, they respond to the shock in the opposite way. Specifically, individual demand for housing initially rises, and continues to increase over time. This is because this group of households is patient enough so that they can afford the losses arising from low anticipated house prices, and enjoy more housing service in the future. Because the losses from the temporary decline in house prices can be compensated by the future gains from enjoying more housing, the individual demand for housing increases. In order to accumulate more housing, patient households forgo some consumption and thus the individual demand for final goods decreases. Since the decrease in demand for housing from impatient households exceeds the increase in demand for housing from patient households, aggregate demand for housing falls initially. This subsequently depresses house prices and housing investment. The depreciation of house prices and the decline in housing investment are reinforced by the continuing decline in aggregate housing demand over time. As we can see from Figure 3, both house prices and housing investment decline significantly in the short run. Note that both house prices and housing investment revert back immediately after an initial decline is a product of the interaction between the individual demand for housing from each group of households. Even if equilibrium aggregate consumption does not initially fall, it will eventually go down as the budget constraint of impatient households tightens over time.

The financial frictions at work during the recession are reflected in the spread between the expected return on assets and the expected cost of deposits. In the model with financial frictions, the spread increases as an outcome of the decline in the bank's net worth. In particular, when the spread rises, the cost of financing capital increases to the extent that debtor firms must downsize the amount of capital investment and reduce its volume of output. Therefore, the magnified declines in capital investment and output can be attributed to the increase in the spread during the recession.

Banks can restore their leverage ratio either by deleveraging or accumulating more net worth. In the process of deleveraging, banks have to increase their net worth. As we can see from Figure 3, an increase in net worth is accompanied by a decline in the spread. So long as the spread is above its trend, net worth cannot immediately revert to it's steady-state level. Throughout the transition path to the long-run trend, the convergence process is slow and it takes a long time for the bank to restore its leverage ratio. Strictly speaking, the change in the spread between expected returns on assets and costs on deposits induced by the financial frictions slows down the pace of recovery in the economy. In this way, our baseline model is successful in it's ability to capture the mechanism through which the deleveraging process slows down the recovery of the economy during the Great Recession, which is consistent with Gertler and Kiyotaki (2010). The main discrepancy of the results generated from the Gertler-Kiyotaki model and our model, in terms of the pace of the recovery process, is that consumption, business investment, output and deposits in our model tend to converge slightly faster than that in the Gertler-Kiyotaki model. The underlying reason for this is mainly due to the substitution effects on houses and consumption goods induced by the financial shocks.[25]

In the baseline model, the effects of the financial shock are amplified and propagated over time by financial factors. Compared to the alternative model without financial frictions, the decline in both GDP and output are more than doubled. Nonresidential investment and capital stock decline by a larger amount than that in the frictionless model. In the housing market, residential investment falls significantly in the short run, and the housing stock falls at a larger magnitude over time, relative to its counterparts. House prices in both the short run and long run decline at a lager magnitude than its counterpart. With financial frictions, the decline in both commercial loans and household loans are significantly larger than their counterparts. Deposits in the short run and medium run do not drop as much as that in the frictionless model. However, it is slightly larger than its counterpart in the long run. Aggregate consumption in the short and medium run does not fall as much as that in the frictionless case because impatient households are more willing to substitute towards consumption goods and away from houses. Without heterogeneity and housing, both deposits and consumption would fall by a larger amount than their counterparts, as illustrated in Gertler and Kiyotaki (2010). Moreover, in the frictionless model the spread does not change over time since the expected returns are equal across assets and deposits. Therefore, the spread does not serve as a barrier to the recovery of the economy from the financial crisis in such a model. Conversely, an increase in the spread is the key in the frictional model and generates a magnified effect, slowing down economic recovery from a financial crisis.

7.2.2 The Bank Liquidity Shocks

Figure 4 plots the impulse responses to an estimated bank liquidity shock. Since an increase in θ_t tightens the bank's incentive constraint, a bank will permit less borrowing (deposits) for any given level of net worth, causing a reduction in loans issued. We use this mechanism to motivate a disruption in bank liquidity.

[25] Gertler and Kiyotaki (2010) do not consider the heterogeneity of households, and exclude houses. In absence of these elements, their model is not able to generate the substitution effects on houses and consumption goods. In this regard, consumption, business investment, output and deposits all decline at a larger magnitude than that in our model.

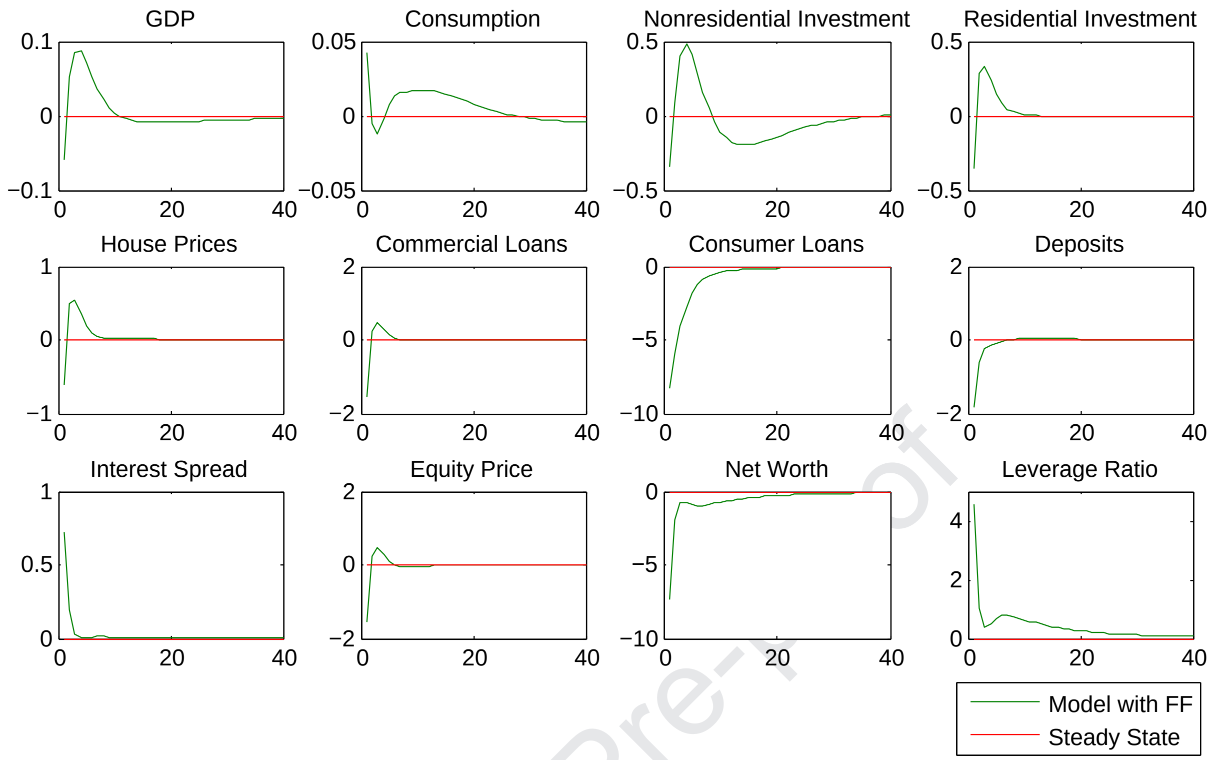

Figure 4: Impulse Responses to a Negative Bank Liquidity Shock

In the baseline model, a disruption in bank liquility makes it difficult for credit-constrained households to borrow from banks, and both housing investment and business investment fall, leading to a decline in asset prices (house prices and equity prices). It then induces a drop in bank net worth that further tightens the bank's incentive constraint, and in this way, the effects of the exogenous shock are magnified and propagated over time. The spread rises due to the decline in bank net worth. When the cost of loans increases, both investment and GDP must fall. The leverage is positively related to the excess return to assets over liabilities, μ_t, and negatively related to θ_t. A rise in the spread is responsible for the increase in the leverage. Similar to the case with the capital quality shocks, financial factors also contribute to the slow recovery of the economy from a financial distress due to the deleveraging process.

Aggregate consumption rises initially due to the offsetting mechanism between the two types of households. On the one hand, a reduction in loans will tighten the budget constraint of impatient households, and hence, consumption falls. One may notice that the effects of the reduction in loans on consumption and housing demand of impatient households are magnified by the collateral constraint. On the other hand, demand for consumption goods by patient households increases over time. As a result, aggregate consumption rises initially in response to the shocks.

In the model without financial frictions, the effects of liquidity shocks on the model dynamics are trivial, and are not of interest. Note that in this case a change in θ_t would not affect the model dynamics since the bank incentive constraint is slack. We omit the impulse responses for this case in Figure 4.

7.2.3 The Housing Preference Shocks

Figure 5 plots impulse responses to the estimated housing preference shock. The housing preference shock is the one type of the housing demand shocks. In the baseline model, a positive housing preference shock drives up the housing demand and house prices, and in turn, raises the borrowing capacity of borrowers (impatient households), since the value of the houses as a collateral rises. Consequently, household loans increase, in which case, it further increases house prices and housing investment. Aggregate consumption rises on impact as a consequence of the increase in the demand for consumption goods from borrowers, even if the demand for consumption goods from savers (patient households) falls.

Since the increase in household loans crowd out commercial loans, the latter falls, causing a decline in business investment. The decline in business investment is responsible for a drop in net worth. In particular, when the demand for capital decreases, equity prices must fall, leading to a drop in net worth. So long as the bank net worth is below the trend, the spread must overshoot the trend. The dynamics of deposits are straightforward. An increase in house prices fundamentally relax the budget constraint of the savers, allowing them to increase deposits.

Output falls initially, and then rises due to the opposite movement between consumption and business investment. GDP rises over time due to the increases in consumption and housing investment, even if business investment falls.

Finally, we turn to the case without financial frictions. In Figure 5, one may find that both business investment and capital stock fall slightly at initial periods, and then immediately revert to increase over time, in which case, it turns out distinct dynamic patterns from what we have observed in the frictional case.

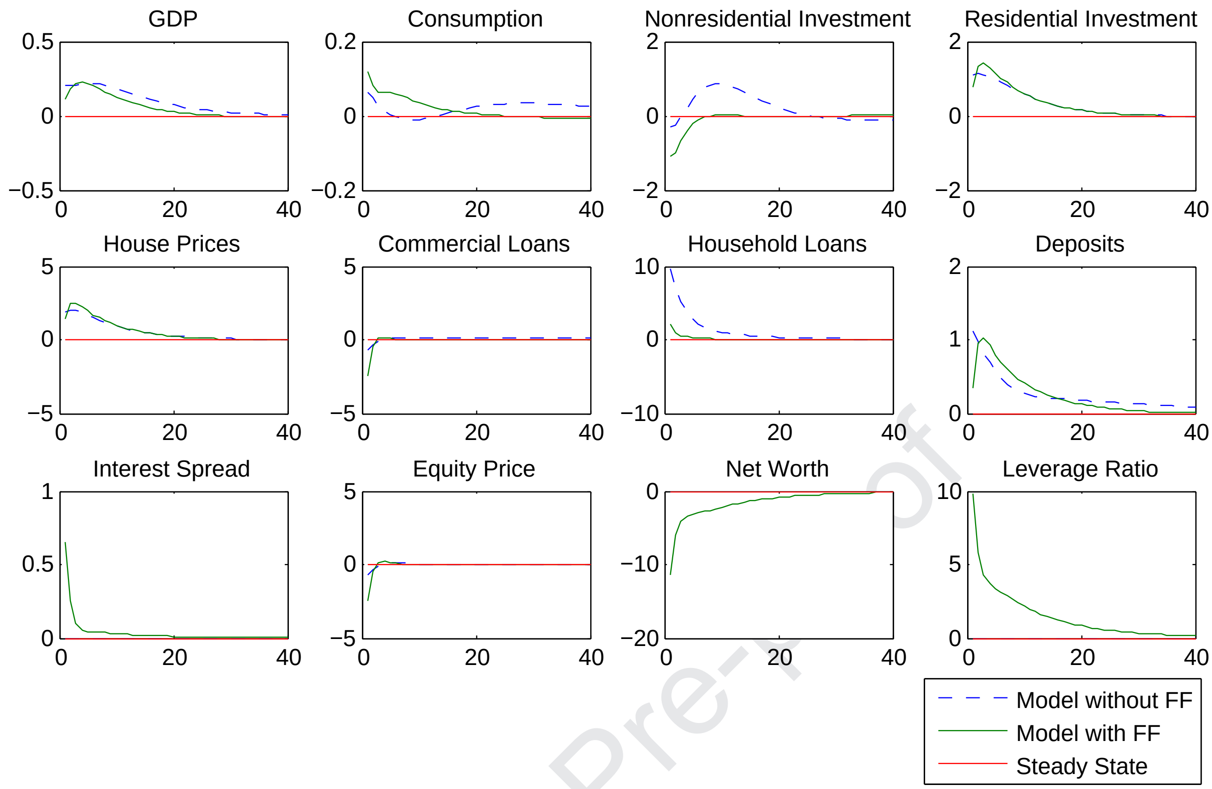

Figure 5: Impulse Responses to a Positive Housing Preference Shock

7.2.4 The Technology Shocks in the Final Goods Sector

Figure 6 plots impulse responses to the estimated technology shock in the final goods sector. A positive technology shock in the final goods sector drives up the expected return to capital, and hence, business investment (see the Nonresidential Investment panel in Figure 6) and output. The response of bank net worth to the shock is magnified in two ways. First, because the bank is highly leveraged, the rise in the expected return to capital raises the value of its assets and, consequently, enhances its net worth by a factor equal to the leverage ratio. Second, the rise in the net worth relaxes the bank's borrowing constraint, causing an accumulation of its assets that further raises the asset prices. Both forces reinforce each other and generate a significant rise in the banks' net worth. With a high value of net worth, the bank can make more loans to its borrowers and accept more deposits from savers. Consequently, more funds are fed into the real economy, stimulating an increase in the capital stock, GDP, and house prices.

We now analyze the responses by each type of household to a positive technology shock in the

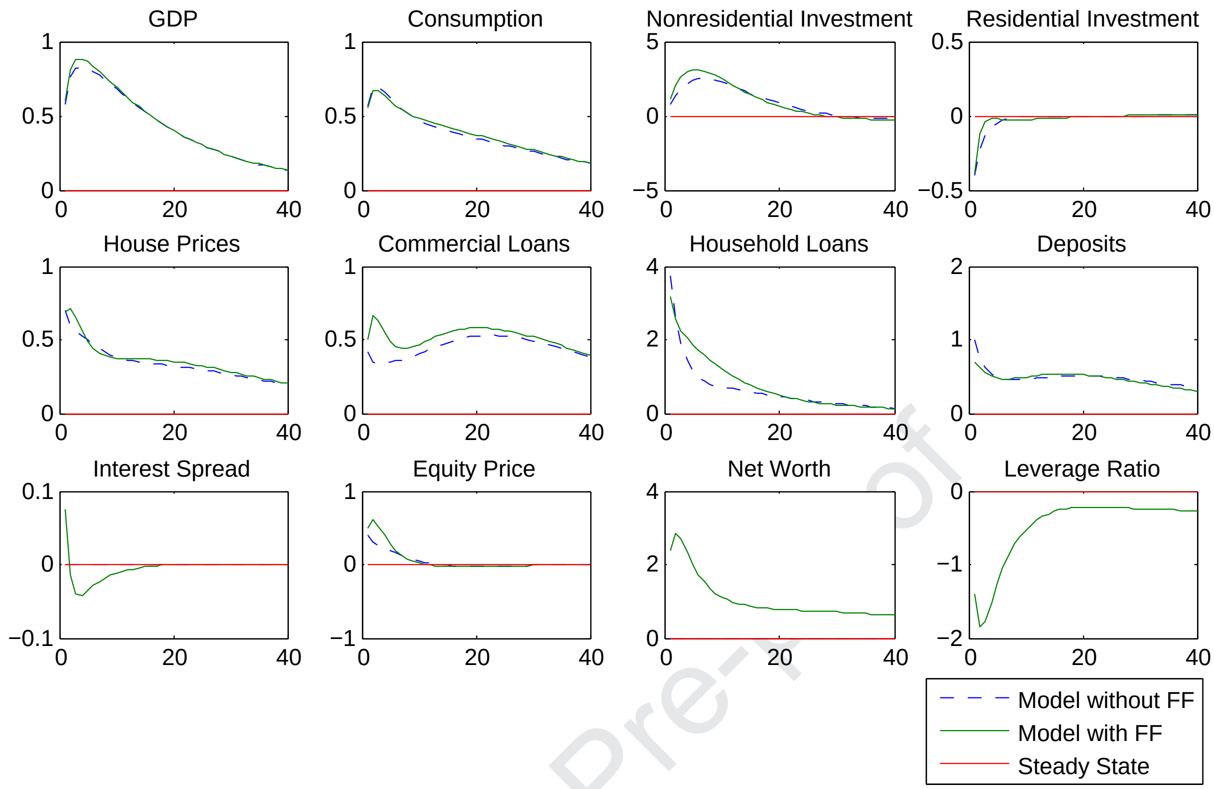

Figure 6: Impulse Responses to a Positive Technology Shock in the Final Goods Sector

final goods sector. For impatient households, the demand for both consumption goods and housing increase over time, thanks to the relaxation of their budget constraints. For patient households, the demand for consumption goods also rises. The demand for housing, however, declines over time due to a dominant substitution effect induced by the high price of houses relative to consumption goods. Overall, aggregate consumption rises and aggregate demand for housing falls. In addition, housing investment declines due to a rise in construction costs (e.g. wages and land rents) and weak demand for housing. But the magnitude of the decline in housing investment exceeds that of the aggregate demand for housing so that house prices rise over time. Note that a positive technology shock in the final goods sector generates a negative correlation between house prices and housing investment, which contradicts the data. The inability of the model with supply-side shocks alone to account for the positive comovement between house prices and housing quantities is along the lines of the existing housing literature such as Davis and Heathcote (2005). This is a standard problem for any model driven by supply-side shocks.

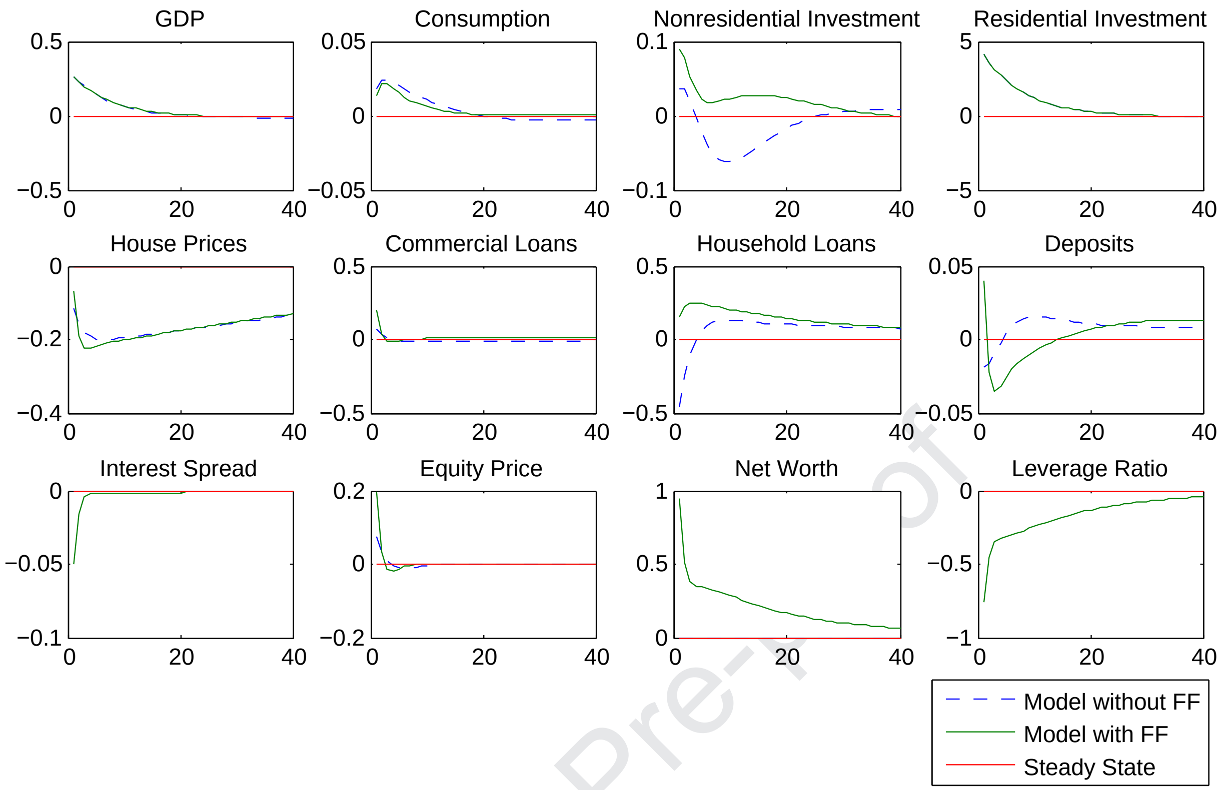

Figure 7: Impulse Responses to a Positive Productivity Shock in the Housing Sector

7.2.5 The Technology Shocks in the Housing Sector

Figure 7 plots impulse responses to the estimated technology shock in the housing sector. A positive technology shock in the housing sector directly drives up the marginal product of labor, and hence, housing investment, thanks to a decline in construction costs. The increase in housing investment is ultimately responsible for a drop in house prices. Consumption rises over time due to the dominant wealth effect induced by the shocks. A positive technology shock in the housing sector also raises business investment, capital stock and output as the demand for consumption goods rises. Therefore, GDP increases over time.

Next, consider the movements of financial variables in this case. The decline in household loans is a product of the low expected house prices. With a low housing price, banks are less willing to issue household loans to their borrowers, given houses as a collateral. Commercial loans increase over time due to a rise in business investment. Recall that asset/capital prices are positively associated with commercial loans and business investment. When both commercial loans and business investment rise, asset/capital prices increase over time, causing a decline in the spread. The reason that net worth rises on impact is quite straightforward. So long as asset prices are above the trend, net worth rises. Last, when the increase in net worth exceeds the increase in assets, bank leverage ratio must fall. Throughout the convergence process, banks effectively releverage since they are building up assets relative to debts.

8 Conclusion

In this paper, we develop and estimate a DSGE model with housing and banking where bank losses initiated by financial shocks can produce a persistent and magnified effect on the housing market and the macroeconomy over the business cycle. The amplification of financial shocks in the model are originated from financial frictions tied to banks and constrained households. These frictions reinforce each other and propagate over time to the business cycles, inducing an amplified effect on the dynamics of financial and housing variables. The dynamics generated by the estimated model are qualitatively consistent with our empirical evidence. These findings suggest that the dynamics of house prices are not only driven by non-financial disturbances such as technology shocks, but also driven by financial disturbances such as capital quality shocks, bank liquidity shocks and housing preference shocks. Any neglect of the importance of these financial shocks in affecting the dynamics of house prices may not provide a better understanding of business fluctuations in the housing market.

The model presented in this paper could be generalized in various directions to address a number of issues associated with housing and financial markets. This paper, however, does not explicitly consider the default of indebted households. One could extend the model by including an endogenous default decision made by credit-constrained households to investigate the extent to which the presence of default risks affects the financial and housing markets (e.g. Ferrante, 2015). Moreover, a large class of DSGE models with housing only is unable to quantitatively account for the comovements between financial and housing variables observed in the data. The model with housing and banking might be a good candidate to address the drawbacks of those previous housing models. It seems to us that addressing these issues in a DSGE model with housing and banking is a sensible thing to do and we leave them for our future research.

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Appendix

A Data and Sources

Real Aggregate Consumption: Real Personal Consumption Expenditures (Seasonal Adjusted, Billions of Chained 2005 Dollars, Source: Table 1.1.6, NIPA, Bureau of Economic Analysis (BEA)).

Real Nonresidential Investment: Real Private Nonresidential Fixed Investment from 1995:Q1 to 2011:Q4 (Seasonally Adjusted, Billions of Chained 2005 Dollars, Source: Table 1.1.6, NIPA, Bureau of Economic Analysis (BEA)). Real Private Nonresidential Fixed Investment from 1973:Q1 to 1994:Q4 is unavailable in the BEA and is calculated by Nominal Private Nonresidential Fixed Investment at base year 2005 multiplied by Quantity Indexes for Real Nonresidential Fixed Investment (Seasonally Adjusted, Index Numbers: 2005=100, Source: Table 5.3.3, NIPA, Bureau of Economic Analysis (BEA)) and divided by quantity index 100 at base year 2005.

Real Residential Investment: Real Private Residential Fixed Investment from 1995:Q1 to 2011:Q4 (Seasonally Adjusted, Billions of Chained 2005 Dollars, Source: Table 1.1.6, NIPA, Bureau of Economic Analysis (BEA)). Real Private Residential Fixed Investment from 1973:Q1 to 1994:Q4 is unavailable in BEA and is calculated by Nominal Private Residential Fixed Investment at base year 2005 multiplied by Quantity Indexes for Real Residential Fixed Investment (Seasonally Adjusted, Index Numbers: 2005=100, Source: Table 5.3.3, NIPA, Bureau of Economic Analysis (BEA)) and divided by quantity index 100 at base year 2005.

Real Gross Domestic Product: The sum of Real Personal Consumption Expenditures, Real Private Nonresidential Fixed Investment and Real Private Residential Fixed Investment (Seasonally Adjusted, Billions of Chained 2005 Dollars, Source: Table 1.1.6, NIPA, Bureau of Economic Analysis (BEA)).

Real House Prices: Freddie Mac House Price Index (Source: FMHPI). The FMHPI only accounts for houses with mortgages within the conforming amount limits, and does not account for jumbo mortgages. The FMHPI series is only available from 1975. To ensure compatibility across all time series used in this paper, we extrapolate the FMHPI series backwards for the period 1973-1974 using the growth rate of the U.S. Census Bureau House Price Index during the same period. The series is deflated by the GDP deflator.

Real Commercial Loans: The sum of C&I Loans and Commercial Real Estate Loans (Source: Assets and Liabilities of Commercial Banks in the United States (Table H8), FRB) divided by the GDP deflator. Commercial Real Estate Loans is only available from 2004:Q3. To ensure compatibility across all time series used in this paper, we extrapolate the series backwards for the period 1973:Q1-2004:Q2 using the growth rate of Total Mortgages to Nonfinancial Business (Source: Table L.102, Flow of Funds Accounts, FRB) during the same period.

Real Household Loans: The sum of Consumer Loans and Residential Real Estate Loans (Source: Assets and Liabilities of Commercial Banks in the United States (Table H8), FRB) divided by the GDP deflator. Residential Real Estate Loans is only available from 2004:Q3. To ensure compatibility across all time series used in this paper, we extrapolate the series backwards for the period 1973:Q1-2004:Q2 using the growth rate of Home Mortgages to Households and Nonprofit Organizations (Source: Table L.101, Flow of Funds Accounts, FRB) during the same period.

Real Net Worth: Residual: Assets less Liabilities (Source: Assets and Liabilities of Commercial Banks in the United States (Table H8), FRB) divided by the GDP deflator.

B Proof of Proposition 1

In this appendix, we give full details in deriving undetermined coefficients for the conjectured value function. We first guess the value function $V_t(s_t, b_t, d_t)$ is linear in (s_t, b_t, d_t),

$$V_t(s_t, b_t, d_t) = \nu_{s,t}s_t + \nu_{b,t}b_t - \nu_{d,t}d_t.$$

Given the flow-of-funds constraint (12), the incentive constraint (13) can be rewritten as

$$V_t \geq \theta_t(n_t + d_t). \tag{B.1}$$

Then we replace V_t in equation (B.1) with equation (16) together with the flow-of-funds constraint (12) to get an alternative expression for the incentive constraint, that is

$$[\theta_t - (\nu_{b,t} - \nu_{d,t})]d_t + (\nu_{b,t} - \frac{\nu_{s,t}}{p_t})p_t s_t \leq (\nu_{b,t} - \theta_t)n_t. \tag{B.2}$$

where equation (B.2) holds with equality if $\lambda_t^b > 0$, and with strict inequality if $\lambda_t^b = 0$. The first order condition (17) implies that

$$\frac{\nu_{s,t}}{p_t} = \nu_{b,t}, \tag{B.3}$$

where the marginal value of equity is always equal to the marginal value of household loans no matter the incentive constraint is binding or not, implying that banks are indifferent between issuing commercial loans to non-financial firms and issuing household loans to impatient households. Moreover, we rewrite the first order condition (18) as follows,

$$\nu_{b,t} - \nu_{d,t} = \frac{\theta_t \lambda_t^b}{1 + \lambda_t^b}. \tag{B.4}$$

Substituting equation (B.3) and (B.4) into the incentive constraint (B.2) yields

$$d_t \leq \frac{1 + \lambda_t^b}{\theta_t}(\nu_{b,t} - \theta_t)n_t. \tag{B.5}$$

Similarly, the conjectured value function (16) together with equation (B.3) and (B.4) yields

$$V_t = \frac{\theta_t \lambda_t^b}{1 + \lambda_t^b}d_t + \nu_{b,t}n_t. \tag{B.6}$$

While substituting equation (B.5) into equation (B.6) to replace d_t, we get an new expression for the value function,

$$V_t = [\lambda_t^b(\nu_{b,t} - \theta_t) + \nu_{b,t}]n_t \tag{B.7}$$

where the term in the bracket is the marginal value of net worth to the ongoing bank. Intuitively, with an additional unit of net worth, the bank can issue an additional household loans to impatient households by obtaining a benefit of $\nu_{b,t}$, and in turn, relax the incentive constraint by $\nu_{b,t} - \theta_t$, which increases the value of the bank by a factor equal to λ_t^b.

Finally, we substitute equation (B.7) for date $t+1$ into the Bellman equation (15) to yield

$$V_t(s_t, b_t, d_t) = E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1} \tag{B.8}$$

with

$$\Omega_{t+1} = 1 - \sigma + \sigma[\lambda_{t+1}^b(\nu_{b,t+1} - \theta_t) + \nu_{b,t+1}] \tag{B.9}$$

where Ω_{t+1} is the marginal value of net worth at date $t+1$.

Given equations (B.3) and (B.4), we can derive an expression for the excess value of returns on assets over deposits,

$$\mu_t = \frac{\nu_{s,t}}{p_t} - \nu_{d,t} = \nu_{b,t} - \nu_{d,t} = \frac{\theta_t \lambda_t^b}{1 + \lambda_t^b}. \tag{B.10}$$

Combining equation (B.10) for date $t+1$ with equation (B.9), we can obtain an new expression for the marginal value of net worth at date $t+1$, that is

$$\Omega_{t+1} = 1 - \sigma + \sigma(\nu_{d,t+1} + \phi_{t+1}\mu_{t+1}) \tag{B.11}$$

with

$$\phi_{t+1} = \frac{\nu_{d,t+1}}{\theta_t - \mu_{t+1}} \tag{B.12}$$

where ϕ_{t+1} is the bank's leverage ratio at date $t+1$.

By applying the method of the undetermined coefficients to equation (B.8), we can easily determine all coefficients to the conjectured value function as follows,

$$\nu_{s,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [Z_{t+1} + (1 - \delta_k) p_{t+1}] \psi_{t+1} \tag{B.13}$$

$$\nu_{b,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^b \tag{B.14}$$

$$\nu_{d,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^d \tag{B.15}$$

Therefore, the conjectured value function V_t is linear in (s_t, b_t, d_t) if and only if the conditions (B.13), (B.14), and (B.15) are satisfied. **QED**

C Dynamic System of the Baseline Model

$$u_{cp,t} = \Gamma_p(\frac{A_{p,t}}{c_{p,t}-\tau_p c_{p,t-1}} - \frac{\beta_p \tau_p A_{p,t+1}}{c_{p,t+1}-\tau_p c_{p,t}}) \tag{C.1}$$

$$1 = \beta_p E_t(\frac{u_{cp,t+1}}{u_{cp,t}} R^d_{t+1}) \tag{C.2}$$

$$q_t = \frac{A_{p,t} j_t}{u_{cp,t} h_{p,t}} + \beta_p E_t[\frac{u_{cp,t+1}}{u_{cp,t}}(1-\delta_h) q_{t+1}] \tag{C.3}$$

$$p_{x,t} = \beta_p E_t[\frac{u_{cp,t+1}}{u_{cp,t}}(p_{x,t+1} + R^x_{t+1})] \tag{C.4}$$

$$w_{pc,t} = \frac{A_{p,t}}{u_{cp,t}}(l_{pc,t}^{1+\epsilon_p} + l_{ph,t}^{1+\epsilon_p})^{\frac{\eta_p-\epsilon_p}{1+\epsilon_p}} l_{pc,t}^{\epsilon_p} \tag{C.5}$$

$$w_{ph,t} = \frac{A_{p,t}}{u_{cp,t}}(l_{pc,t}^{1+\epsilon_p} + l_{ph,t}^{1+\epsilon_p})^{\frac{\eta_p-\epsilon_p}{1+\epsilon_p}} l_{ph,t}^{\epsilon_p} \tag{C.6}$$

$$u_{ci,t} = \Gamma_i(\frac{A_{p,t}}{c_{i,t}-\tau_i c_{i,t-1}} - \frac{\beta_i \tau_i A_{p,t+1}}{c_{i,t+1}-\tau_i c_{i,t}}) \tag{C.7}$$

$$\frac{A_{p,t} j_t}{u_{ci,t} h_{i,t}} + \beta_i E_t[\frac{u_{ci,t+1}}{u_{ci,t}}(1-\delta_h) q_{t+1}] = q_t - \lambda_{i,t} m_t E_t(\frac{q_{t+1}}{u_{ci,t} R^b_{t+1}}) \tag{C.8}$$

$$1 = \beta_i E_t(\frac{u_{ci,t+1}}{u_{ci,t}} R^b_{t+1}) + \frac{\lambda_{i,t}}{u_{ci,t}} \tag{C.9}$$

$$w_{ic,t} = \frac{A_{p,t}}{u_{ci,t}}(l_{ic,t}^{1+\epsilon_i} + l_{ih,t}^{1+\epsilon_i})^{\frac{\eta_i-\epsilon_i}{1+\epsilon_i}} l_{ic,t}^{\epsilon_i} \tag{C.10}$$

$$w_{ih,t} = \frac{A_{p,t}}{u_{ci,t}}(l_{ic,t}^{1+\epsilon_i} + l_{ih,t}^{1+\epsilon_i})^{\frac{\eta_i-\epsilon_i}{1+\epsilon_i}} l_{ih,t}^{\epsilon_i} \tag{C.11}$$

$$b_t = m_t E_t(\frac{q_{t+1} h_{i,t}}{R^b_{t+1}}) \tag{C.12}$$

$$c_{i,t} + q_t h_{i,t} + R^b_t b_{t-1} = w_{ic,t} l_{ic,t} + w_{ih,t} l_{ih,t} + q_t(1-\delta_h) h_{i,t-1} + b_t \tag{C.13}$$

$$w_{pc,t} = \alpha(1-\mu_c)\frac{Y_t}{l_{pc,t}} \tag{C.14}$$

$$w_{ic,t} = (1-\alpha)(1-\mu_c)\frac{Y_t}{l_{ic,t}} \tag{C.15}$$

$$Z_t = \mu_c \frac{Y_t}{k_t} \tag{C.16}$$

$$Y_t = (A_{c,t}(l_{pc,t}^{\alpha} l_{ic,t}^{1-\alpha}))^{1-\mu_c} K_t^{\mu_c} \tag{C.17}$$

$$w_{ph,t} = \alpha(1-\mu_h)\frac{q_t I_{h,t}}{l_{ph,t}} \tag{C.18}$$

$$w_{ih,t} = (1-\alpha)(1-\mu_h)\frac{q_t I_{h,t}}{l_{ih,t}} \tag{C.19}$$

$$R^x_t = \mu_h \frac{q_t I_{h,t}}{x_{t-1}} \tag{C.20}$$

$$I_{h,t} = (A_{h,t}(l_{ph,t}^{\alpha} l_{ih,t}^{1-\alpha}))^{1-\mu_h} \tag{C.21}$$

$$\Lambda_{t,t+1} = \beta_p \frac{u_{cp,t+1}}{u_{cp,t}} \tag{C.22}$$

$$p_t = 1 + (\frac{I_{k,t}}{I_{k,t-1}} - 1)^2 + 2(\frac{I_{k,t}}{I_{k,t-1}} - 1)\frac{I_{k,t}}{I_{k,t-1}} - 2\Lambda_{t,t+1}(\frac{I_{k,t+1}}{I_{k,t}} - 1)(\frac{I_{k,t+1}}{I_{k,t}})^2 \tag{C.23}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma(\nu_{d,t+1} + \phi_{t+1}\mu_{t+1}) \tag{C.24}$$

$$\nu_{b,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^b \tag{C.25}$$

$$\nu_{d,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^d \tag{C.26}$$

$$\nu_{s,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} \psi_{t+1}(Z_{t+1} + (1-\delta_k)p_{t+1}) \tag{C.27}$$

$$\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1}(R_{t+1}^k - R_{t+1}^d). \tag{C.28}$$

$$R_{t+1}^k = \psi_{t+1} \frac{Z_{t+1} + (1-\delta_k)p_{t+1}}{p_t} \tag{C.29}$$

$$R_{t+1}^k = R_{t+1}^b \tag{C.30}$$

$$p_t S_t + B_t = N_t + D_t \tag{C.31}$$

$$p_t S_t + B_t = \phi_t N_t \tag{C.32}$$

$$\phi_t = \frac{\nu_{d,t}}{\theta_t - \mu_t} \tag{C.33}$$

$$N_t = (\sigma + \xi)\{(Z_t + (1-\delta_k)p_t)\psi_t S_{t-1} + R_t^b B_{t-1}\} - \sigma R_t^d D_{t-1} \tag{C.34}$$

$$Y_t = C_t + (1 + (\frac{I_{k,t}}{I_{k,t-1}} - 1)^2) I_{k,t} \tag{C.35}$$

$$I_{h,t} = H_t - (1-\delta_h)H_{t-1} \tag{C.36}$$

$$S_t = I_{k,t} + (1-\delta_k)K_t \tag{C.37}$$

$$C_t = c_{p,t} + c_{i,t} \tag{C.38}$$

$$H_t = h_{p,t} + h_{i,t} \tag{C.39}$$

$$K_{t+1} = \psi_{t+1}(I_{k,t} + (1-\delta_k)K_t) \tag{C.40}$$

$$\ln \psi_t = \rho_k \ln \psi_{t-1} + u_{k,t} \tag{C.41}$$

$$\ln A_{c,t} = \rho_c \ln A_{c,t-1} + u_{c,t} \tag{C.42}$$

$$\ln A_{h,t} = \rho_h \ln A_{h,t-1} + u_{h,t} \tag{C.43}$$

$$\ln A_{p,t} = \rho_p \ln A_{p,t-1} + u_{p,t} \tag{C.44}$$

$$\ln m_t = (1-\rho_m)\ln m + \rho_m \ln m_{t-1} + u_{m,t} \tag{C.45}$$

$$\ln j_t = (1-\rho_j)\ln j + \rho_j \ln j_{t-1} + u_{j,t} \tag{C.46}$$

$$\ln \theta_t = (1-\rho_\theta)\ln \theta + \rho_\theta \ln \theta_{t-1} + u_{\theta,t} \tag{C.47}$$

Highlights

We develop a DSGE model to study the transmission of various financial shocks.
Financial frictions tied to banks and households interact each other over time.
Financial shocks are critical to the dynamics of housing and macroeconomic variables.
The dynamics of house prices is associated with the banks' balance sheets.
Capital quality shocks generate a decline in household loans, house prices and output.