Design a High Efficiency and Low Ripple BLDC Motor Based on Multi-Objective Optimization Methods

Pouria Karimi Shahri¹, Vahid Izadi², and Amir H. Ghasemi³

Abstract—In this paper, we used an adaptive multi-objective optimization to increase the efficiency and decrease the torque ripple frequency of a BLDC motor by changing the dimensions of the stator slots. Torque ripple usually happens when there is a variation in torque production and it causes unwanted vibrations and speed fluctuations. Simulations are used to demonstrate the effectiveness of each parameter in both efficiency and torque ripple as our objectives so we can have several design points and based on these points it is possible to reach to a mathematical formula in which describes the optimization problem. In order to get an explicit formula from true unknown responses, different meta-modelling methods were used and to solve the proposed optimization problem, Genetic Algorithm (GA) method was utilized in order to get pareto frontiers and compare the results. Also in this paper, the accuracy of each model was checked by measuring its statistical parameters. The numerical results for the proposed optimization result show the high accuracy for the proposed approach with CS(3,0) and CS(3,1) response function methods.

I. INTRODUCTION

Nowadays, using high-efficiency electromechanical machines are in the center of considerations from their energy consumption aspect. The brushless DC motors (BLDCMs) are the best candidates in the many spaces, industrial and domestic applications [1]–[4]. These types of electrical motors have higher efficiency, torque density, and lower maintenance for induction motors. There are many effective parameters from the aspect of the cost, efficiency, and input-output characteristics in the BLDC motor design process. To reach a nominal performance for a low cost motor, it is essential to identify the effect of each parameter and variable in the final product.

In a general view, there are two sections that have a role in the motor performance: electrical section and electromechanical section. The electrical part contains the controller and driver parts. In most references to improve the input-output characteristics of the BLDC motors, the main focus of the researches is concentrated on the electrical section. In the motor driving methods, there is some amount of leakage, vibration, and current harmonics may be considered as other criteria for optimization.

Then to reach the higher efficiency and lower amount of torque fluctuations as objective functions, it is better to concentrate on the design optimization of the electromechanical section. In some applications, in addition to the mentioned objective function, the cost, volume, cogging torque, flux leakage, vibration, and current harmonics may be considered as other criteria for optimization.

As it is stated before, based on the complexity in the design parameters and their relationship to the objective functions, the motor design will be a time-consuming procedure. There are several literature on the BLDC motor design optimization [17]–[26]. A deterministic global-optimization method is used to design of a slotless permanent magnet motor based on geometry mentioning [17]. In most cases in which there are multiple objective functions in an opti-
mization problem, the GA-based optimal has been applied [13], [18], [19]. Bianchi et al. [18] provided a comparative study between GA, classical gradient-based, and direct search optimization approaches. As it is obvious in these works, the evaluation and verification step in the design and optimization process is based on a package of raw data or category of simplified equations. In this paper, a systematic approach based on adaptive multiobjective optimization by using meta-modeling methods is presented. In this approach, the first number of samples will be used as an input for meta-modeling. The samples are generated based on the FEA analysis for different stator slot dimensions. In the meta-modeling stage, different polynomial regression (PR) and radial basis function (RBF) interpolation are created by using the full factorial design (FFD). The mathematical relation between the stator slots dimensions and the BLDC motor efficiency and torque ripple is a nonlinear equation. The response surface methodology (RSM) is appropriate for creating global models for linear systems. The created models are appropriate for the gradient-based optimization method because of their simplicity. On the other hand, radial basis functions (RBFs) are useful tools for creating models for highly nonlinear responses. The augmented RBF models provide more accurate data for non-gradient base multi-optimizations methods, but they are computationally more expensive. Fang et al. in [27], according to the study, which is performed on RBFs versus RSM, shows the accuracy of each method for different test functions. In this paper, both the meta-modeling method are used to generate sample data for the optimizing step. In this paper, the non-dominated sorting genetic algorithm (NSGA) and feasible sequential quadratic programming (FSQP) are used to calculate the Pareto frontier. The NSGA approach, because of reducing the number of redundant comparisons in the algorithm, is much faster than the conventional genetic algorithm [28], [29].

The outline of this paper is as follows. Section II presents the basics of the design of the experiment and standard formulation. Section III presents the concept of meta-modeling in design optimization of the BLDC motor and its different methods. Section IV presents the FSQP and NSGA-II methods in the adaptive MOO. In section V the numerical results are presented, and section 5 is the conclusion.

II. DESIGN OF EXPERIMENT AND STANDARD FORMULATION

To define the basic parameters in the BLDC motor design according to the application, the characteristic values (power-speed, power-torque, and torque-speed) will be defined. After the determination of the rated characteristic values, which is shown in Table I, the main dimension of the motor and the electromagnetic load can be defined. In figure 1, the general design process for a BLDC demonstrated [30]. It must be considered that in this paper, the optimization is done after some design process for core and wire materials, height, and diameter of the rotor and stator. The area of interest in this paper is defined in the design flowchart by a red rectangle.
Table I
THE GENERAL CHARACTERISTIC FOR THE SIMULATED BLDC MOTOR

<table>
<thead>
<tr>
<th>System parameters</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage</td>
<td>V</td>
<td>13.5</td>
</tr>
<tr>
<td>Rated output power</td>
<td>W</td>
<td>110</td>
</tr>
<tr>
<td>Rated torque</td>
<td>Nm</td>
<td>0.35</td>
</tr>
<tr>
<td>Rated speed</td>
<td>rpm</td>
<td>3000</td>
</tr>
<tr>
<td>Rated efficiency</td>
<td>%</td>
<td>88</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>rpm</td>
<td>5000</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>Nm</td>
<td>1.2</td>
</tr>
<tr>
<td>Cogging torque</td>
<td>Nm</td>
<td>0.238</td>
</tr>
<tr>
<td>Outer diameter of rotor</td>
<td>mm</td>
<td>48</td>
</tr>
<tr>
<td>Stack length</td>
<td>mm</td>
<td>10</td>
</tr>
<tr>
<td>Number of slot</td>
<td>Slots</td>
<td>12</td>
</tr>
<tr>
<td>Diameter of coil</td>
<td>mm</td>
<td>0.7</td>
</tr>
<tr>
<td>Number of turns</td>
<td>Turns</td>
<td>15</td>
</tr>
<tr>
<td>Number of pole</td>
<td>Pole</td>
<td>14</td>
</tr>
<tr>
<td>Permanent magnet</td>
<td>–</td>
<td>N42UH</td>
</tr>
<tr>
<td>Length of air-gap</td>
<td>mm</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The general characteristic for the simulated BLDC motor

The efficiency $\mu$ is defined by output power over input power. In particular:

$$\mu = \frac{T_e \times n_{Rotor}}{V_{Bus} \times I_{Bus}}$$ (1)

Where $T_e$ is the electro-mechanical torque, $n_{Rotor}$ is the speed of the rotor, $V_{Bus}$ is the bus voltage, and $I_{Bus}$ is the bus current. In addition, the torque ripple in a BLDC motor is defined by:

$$T_{Ripple} = T_{6c} \cos (6wt) + T_{6s} \sin (6wt) + T_{12c} \cos (12wt) + \cdots$$ (2)

Where $T_{6c}$ and $T_{6s}$, $i = 1, 2, \ldots$ are harmonic torque, $w$ is the speed of the rotor and $t$ is the time. In order to solve an optimization problem, it is necessary to write its standard formulation and define equality and inequality constraints. The standard formulation of the problem is as follows:

$$\text{Min.} \quad f_1 = -\mu = \frac{T_e \times n_{Rotor}}{V_{Bus} \times I_{Bus}}$$

$$\text{S.t.} \quad g_1 = \frac{2B_{S1}}{B_{S2}} = 1$$

$$g_2 = \frac{4H_{S1}}{H_{S2}} = 1$$

$$B_{S0}, B_{S1}, B_{S2}, H_{S0}, H_{S1}, H_{S2} > 0$$ (3)

According to the equality constraints, we can reduce the number of design parameters to four. Then, in the optimization process, the design parameters are $B_{S0}, B_{S1}, H_{S0}$ and $H_{S1}$ as it is shown in figure 2. The next step is to determine the design points, which has several methods like Full Factorial Design (FFD), Central Composite Design (CCD), Latin hyperCube Sampling (LHS), Taguchi Orthogonal Array. In this paper, FFD was chosen because it is the complete method for determining the design points. The sample size grows exponentially as follow:

$$N_p = N^M$$ (4)

$N$ is the number of design levels, and $M$ is the number of variables. In the following step, we have to use the design points created in (1) to make a function that approximates the system responds in meta modelling.

III. META MODELLING

Meta-modeling allows to create an explicit, approximate function of true unknown responses by using different design variables. In this paper, we have implemented two different methods to reach the approximation functions: Polynomial Regression and Radial Basis Function. In the "Polynomial Regression" (PR) method both linear and quadratic forms were applied. Specifically, we defined

$$f(x) = \alpha_0 + \sum_{j=1}^{m} \alpha_j x_j + \sum_{j=1}^{m} \alpha_{jj} x_j^2 + \sum_{j=1}^{m-1} \sum_{i=j+1}^{m} \alpha_{ij} x_j x_i$$ (5)

where $x_j$ is the design variable, $\alpha_j$ is the unknown coefficient of the polynomial and $m$ is the number of design points. The first two terms are for linear polynomials, the third term is for quadratic polynomial without interaction, the last term is for a quadratic form with pair-interaction. Also, to determine the least-square estimator of the true coefficients we have:

$$\{\alpha\} = \left([X]^T [X] \right)^{-1} \left([X]^T \{f\} \right)$$ (6)

where $X$ is the matrix of the variables (design matrix) and $f$ is the respond vector. To check the accuracy of this model, it is necessary to measure the major statistical parameters such as mean root square, coefficient of multiple determination, and the total sum of squares. The total sum of squares (SST) and the sum of squared errors (SSE) was measured as follow:

$$\text{SST} = \sum_{j=1}^{m} (f_j - \hat{f})^2$$ (7)

$$\text{SSE} = \sum_{j=1}^{m} (f_j - \hat{f})^2$$ (8)

in which, $f_j$ is the value of the function in each design point, $\hat{f}_j$ is the value of function which was calculated from the
polynomial in each design point and \( \bar{f} \) is the mean value of the function. Moreover, root mean square error (RMSE) is:

\[
\text{RMSE}(PR) = \sqrt{\frac{\text{SSE}}{m - p - 1}}
\]

(9)

Where \( p \) is the number of non-constant terms. And for evaluating the coefficient of multiple determination we have:

\[
R^2 = 1 - \frac{\text{SSE}}{\text{SST}} = 0 \leq R \leq 1
\]

(10)

The next model, which is used in this paper to reach out to the approximate function, is "Radial Basis Function" (RBF). This method is useful for both large and small numbers of data points in both linear and highly nonlinear responses. Also, there is no error at design points, so it is required to check the model accuracy by using off-design points (validation points). The standard formulation for RBF is:

\[
f(x) = \sum_{j=1}^{m} \psi_j \phi(||x - x_j||)
\]

(11)

where \( m \) is the number of design points, \( x \) is a vector of design parameters, \( \phi \) is the basis function and \( \psi_j \) is the coefficient for each basis function.

\[
\begin{bmatrix}
  f_1 \\
  f_2 \\
  \vdots \\
  f_m
\end{bmatrix} =
\begin{bmatrix}
  \phi(||x_1 - x_1||) & \cdots & \phi(||x_1 - x_m||) \\
  \phi(||x_2 - x_1||) & \cdots & \phi(||x_2 - x_m||) \\
  \vdots & \ddots & \vdots \\
  \phi(||x_m - x_1||) & \cdots & \phi(||x_m - x_m||)
\end{bmatrix}
\begin{bmatrix}
  \psi_1 \\
  \psi_2 \\
  \vdots \\
  \psi_m
\end{bmatrix}
\]

(12)

Some of the most commonly used basis functions are linear, cubic, Gaussian, RBF-CS30 and RBF-CS31. The basis functions used in our problem are as follow:

- Linear: \( \phi(t) = t \)
- Cubic: \( \phi(t) = t^3 \)
- Gaussian: \( \phi(t) = \sqrt{2\pi} \cdot \frac{e^{-t^2/2}}{c^2}, \quad 0 < c \leq 1 \)
- RBF-CS30: \( \phi_{3,0}(t) = (1 - t)^7(5 + 35t + 101t^2 + 147t^3 + 101t^4 + 35t^5 + 5t^6) \)
- RBF-CS31: \( \phi_{3,1}(t) = (1 - t)^6(6 + 36t + 82t^2 + 72t^3 + 30t^4 + 5t^5) \)

For calculating the coefficients we have:

\[
\{ \psi \} = [A]^{-1} \{ f \}
\]

(13)

where \( A \) is the matrix of basis function in each design point and \( f \) is the respond vector. As it was discussed earlier, it is obligatory to check the accuracy and measure the root mean square error of the model just with the help of validation points.

\[
\text{RMSE}_{(RBF)} = \sqrt{\frac{\sum_{j=1}^{m} (\hat{f}_j - f_j)}{m}}
\]

(14)

Where \( m \) is the number of off-design points, \( f_j \) is the value of the true function in each validation point, and \( \hat{f}_j \) is the function value measured from RBF model in each validation point. In this paper, we took advantage of an optimization software called "HiPPO" [27] to attain a function for our model. In this software, after adding the design points and indicating the design method, several design variables, and design levels, we can reach to the approximation function and its statistical parameters. It must also be noted that, by using adaptive multi-objective optimization, it is requisite to add new sample points, which were the preceding validation points to the preliminary design points, and get a new function by replicating the meta-modeling section.

IV. MULTI OBJECTIVE OPTIMIZATION

The optimization problem we have in this paper contains two objectives, which are the efficiency of BLDC motor and its torque ripple. MOO can be represented in mathematical form as follow:

\[
\begin{align*}
\text{Min} & \quad [f_1(x), f_2(x), \ldots, f_n(x)] \\
\text{s.t.} & \quad c_i(x) \leq 0, \quad i = 1, \ldots, m \quad \text{for} \quad x \in M \text{ and } n > 1.
\end{align*}
\]

(15)

In which \( x \in M \) and \( n > 1 \), \( x \) is the solution, \( M \) is the feasible set and \( n \) is the number of objective constraints. There are several methods for solving MOO based on the requirements of the optimization. In several approaches such like simplex method and evolutionary method, which are the subclass of direct search method, we only need the design points to optimize the problem. However, gradient-based methods such as feasible sequential quadratic programming (FSQP), modified newton method, and conjugate gradient method that require the explicit form of the derivatives to solve the MOO problems. In this paper, we utilized a genetic algorithm (GA), which is one of the non-gradient based methods. GA robust in locating global optimas and allows us to synchronously obtain multiple solutions. Additionally, we used FSQP which is an iterative method for constrained nonlinear optimization and has faster convergence in comparison to non-gradient based methods. FSQP can also be used as a merit function that helps to avoid using penalty function. The reason which makes FSQP computationally expensive is that FSQP needs to solve QP sub-problems at each repetition. In particular,

\[
\begin{bmatrix}
H_k \\
\text{diag}(\eta^k)\nabla g(x^k)^T \\
\nabla^2 \text{diag}(\theta^k)
\end{bmatrix}
\begin{bmatrix}
\lambda \\
d \\
c
\end{bmatrix}
= \begin{bmatrix}
-\nabla f(x^k) \\
0 \\
\theta^k
\end{bmatrix}
\]

(16)

where \( g \) is the inequality constraint function, \( \lambda \) is a vector of non-negative Lagrange multiplier estimates, \( H_k \) is the Hessian of Lagrangian and is positive definite and \( c \in R^m \). For each \( i \) have we:

\[
\eta^k_i = \frac{\eta_i}{\sqrt{\eta_i^2 + (\mu_i^k)^2}} + 1 \quad \text{and} \quad \theta^k_i = \left(1 - \frac{\mu_i^k}{\sqrt{\eta_i^2 + (\mu_i^k)^2}}\right)^{0.5}
\]

Another optimization technique that was utilized in this paper is the Non-dominated Sorting Genetic Algorithm (NSGA-II), which is one of the second generation techniques in evolutionary algorithms family. This method is a search-based method that starts with an initial set of solutions and updates the set with the use of non-dominated solutions and finally improves the solutions in the updated set to approach Pareto frontier [31].
V. OPTIMIZATION RESULTS

In the geometry design process of a BLDC motor, the design parameters $B_s_0, B_s_1, H_s_0$ and $H_s_1$ range are [0.1, 2], [2, 12], [0.1, 3] and [0.1, 3] mm respectively. For the FFD, two and three-level are selected for the design parameters in the mentioned ranges, which generate 16 and 81 sample points. In the first trial, the 16 design points and its validation points in the ANSYS MAXWELL construct the corresponding torque ripple and efficiency value. To extract an explicit response function, the design points and the corresponding simulation results are fed to the HiPPO [27] application. Then, by the GimOPT optimization package [29] based on the NSGA-II algorithm, the Pareto frontier of different response functions is specified. The procedure for the 81 sample points and its validation points is the same as the previous one in an adaptive approach, which is explained in sections II to IV.

Fig. 3. Numerical optimization results by using linear function meta-modelling method for 16 and 81 design points

Fig. 4. Numerical optimization results by using linear regression method for 16 and 81 design points

Fig. 5. Numerical optimization results by using Cubic function meta-modelling method for 16 and 81 design points

Fig. 6. Numerical optimization results by using CS(3,0) meta-modelling method for 16 and 81 design points

Fig. 7. Numerical optimization results by using CS(3,1) meta-modelling method for 16 and 81 design points

By using the CS(3,0) and CS(3,1) methods for the meta-modeling step, the Pareto frontier has the same shape for two and three FFD levels. In figures 6 and 7, it is shown that the Pareto frontier is approximately convex, and the optimization results are valid when the FFD levels are low. The ranges for the motor efficiency and torque ripples are [92.85 93.20] percent and [3 7.5] percent, respectively. According to the extracted results, the proposed adaptive MOO approach shows the effectiveness of each meta-modeling method for the BLDC motor design process.

VI. CONCLUSION

This paper presents an adaptive multi-objective optimization in the BLDC motor design process to reach the maximum efficiency and minimum torque ripple. The design criteria are the geometrical dimension of the slots in the stator.
In the design of the experiment step, two and three levels of design points are used in FFD. In the optimization level, the NSGA-II generates the Pareto frontier. The convexity and variation of the target response function is the main clue in improving the adaptive approach. By adding the validation points to the initial points, the final results are more accurate for the higher level of metamodeling. The numerical results for the proposed optimization result show the high accuracy for the proposed approach with CS(3,0) and CS(3,1) response function methods.

REFERENCES


