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Optimal deployment of construction equipment using linear programming with fuzzy coefficients

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Abstract

Decisions made by the experts in the construction industry are usually approximate and contain some sort of imprecision. Classical linear programming (LP) model optimize the decision making situation in a crisp environment. It is difficult to get an optimum decision with imprecise information of the project environment using LP. In the construction industry, identifying optimum number of construction pieces of equipment require experts' knowledge. When certain degree of flexibility needs to be incorporated in the given model to get more realistic results, fuzzy LP is used. But when the parameters on constraints and objective function are in a state of ambiguity then the extension principle is best suited, which is based on personal opinions and subjective judgments. The objective of this paper is to identify the optimum number of pieces of equipment required to complete the project in the targeted period with fuzzy data. A realistic case study has been considered for optimization and LINGO6 has been used to solve the various non-linear equations.

Keywords: Fuzzy sets; Fuzzy numbers; Fuzzy linear programming; Extension principle; Flexibility; Membership function

1. Introduction

Decision making in construction industry is very complex and requires deep knowledge of various construction management techniques. Operations Research (OR) techniques are widely used under such circumstances through appropriate mathematical models. Of all the models of OR Linear Programming (LP) is widely used in the construction industry. In LP models, all the information pertaining to the problem is expressed in terms of linear constraints on the decision variables where the data is precise. Many project managers arrive at feasible decisions using this model.

The construction industry is clearly affected by market conditions, i.e. by ups and downs in construction activity and by the size and the type of the construction projects undertaken. It is also affected by technological innovation in fields such as materials, metallurgy, mechanical systems, electronic sensing and hydraulic controls. The industry focuses on the continuous improvement of its products by introducing advanced technology [1]. In addition, the success of any construction project depends on the efficiency and economy achieved in the construction phase of the project. The economy of the project is dependent on accurate and elaborate analysis in early stages of construction. But in real project, activities must be scheduled under limited resources, such as limited crew sizes, limited equipment amounts, and limited materials [2]. The presence of large number of interacting variables creates a problem for optimization. Decisions are mainly based on the conceptual understanding of the project by the experts and are usually vague. Therefore, consideration of imprecise and vague information becomes an important aspect in the decision making process. In view of uncertain environment prevailing in the construction industry, the ability to arrive at an optimal decision is most important for its success. Hence, decisions in the construction industry are to be taken only after evaluating the feasibility of an alternative with respect to various criteria affecting its outcome.

The traditional quantitative methods of assessing the feasibility of an alternative such as payback period, rate of return, and benefit cost analysis evaluate the project from the aspect of monitory costs and benefits. But many

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non-quantitative factors and approximate numbers such as availability of labor, weather conditions, and number of equipments also influence the construction project. The above methods fail to incorporate the necessary qualitative parameters and uncertainty in decision making and thus it is difficult to get an optimum decision in construction industry for optimal deployment of machinery.

These uncertainties can be accommodated into the analysis using Artificial Intelligence techniques such as fuzzy sets, neural networks, and expert systems. The successful application of fuzzy logic reflects the true situation of the real world, where human thinking is dominated by approximate reasoning. Hence to obtain optimality, hybrid optimization techniques are used for incorporating flexibility in decision making. Fuzzy LP makes it possible to accommodate these intangible factors in a most systematic way. The objective function is characterized by its membership value and so are the constraints. In fuzzy LP, the decision maker establishes a satisfaction criterion rather than just maximizing or minimizing the objective function. Here, each of the constraints is modeled as a fuzzy set with their respective membership values.

The aim of this paper is to introduce the approximate numbers into the analysis for optimal decisions. This is done by incorporating flexibilities in the coefficients of the objective function and constraints for an optimal value. The approach described in this paper is intended to illustrate the practicability of applying fuzzy LP with fuzzy parameters to civil engineering problems and the potential advantages of the resultant information.

2. Construction equipment

Construction industry comprises of broad range of equipment which include scrapers, graders, hydraulic excavators, trenchers, pipe layers, etc. Depending upon the type and nature of the construction jobs, various equipments and tools are required at different point of time during the execution period. These equipments can be accommodated by hiring, buying or by transferring from other sites. It is important to estimate exactly, the number of equipments to be bought, hired and number of equipments that can be adjusted from the other sites. Normally, experts' qualitatively judge the number of equipments required and hence there is every possibility that the estimated numbers may increase or decrease at the site. Optimally deploying these equipments, preparing an equipment schedule or equipment calendar is an important task of the project manager, such that the construction manager may have no difficulty in arranging the equipments for the purpose at the right time and the work will not be held up because of lack of any equipment. It must be remembered that non-availability of the appropriate equipment or extra idle equipments/tools on the site may lead to financial loss and delays. Hence, the knowledge of various equipments and their usage on the site is necessary and proper planning of them will always fetch good results. The number and the capacity of the equipment is entirely dependent on the nature and the size of the project.

3. Literature review

In construction industry, optimal deployment of machinery plays a significant role. Even though conventional quantitative techniques are efficient enough for getting optimal decisions, they have their own drawbacks. Fuzzy set theory was developed by Zadeh in 1965 for analyzing the decision problems involving fuzzy information. Since then, more than 5000 publications have highlighted the concept and diversified the use of fuzzy set theory.

Bellman and Zadeh [3] developed a decision theory based on fuzzy goals and constraints. In their opinion decision is the confluence of fuzzy goals and Constraints. Zadeh [4] outlined the rules of fuzzy set interpretation of linguistic hedges. He presented systematic conversion of qualitative factors into membership grades for decision analysis. Sasikumar and Mujumdar [5] stated that the imprecisely defined goals and constraints are represented as fuzzy sets in the space of alternatives. Ayyub and Haldar [6] developed a method for estimating the duration of construction activities based on fuzzy set models, and the factors affecting the activity duration. In subsequent years, decision methodologies are developed for selecting and designing construction strategies using approximate reasoning. Wang et al. [7] have evaluated a competitive tendering methodology using fuzzy set theory. Lorterapong [13] proposed the fuzzy network scheduling (FNET) model in which a fuzzy heuristic method was developed to solve the resource constraint project-scheduling problem under uncertainty. Kumar et al. [8] applied fuzzy set theory to working capital requirement. Skibniewski and Armijos [9] adopted LP approach to construction equipments and labor assignments. Mohan [10] used fuzzy LP for optimal crop planning for irrigation system dealing with the uncertainty and randomness for the various factors affecting the model.

Tanaka and Asai [11] have formulated a fuzzy LP problem and considered the ambiguity of parameters. Cross and Cabello [12] applied fuzzy set theory to optimization problems, where multiple goals exist. They have solved a multi-objective LP problem with fuzzy parameters for borrowing/lending problem. It is found that several methods have been suggested for including non-quantitative variables into the decision making process. But very few people have incorporated the complete fuzziness in to the problem. A civil engineering problem comprise mostly of complete fuzzy data, which have to be incorporated to arrive at optimal decisions.

In this paper, the scope has been expanded to include applications in civil engineering projects where optimal equipment allocation is required with ambiguity for the number of equipments to be bought or rented in the construction industry. The approach described in this paper illustrates the practical applications of fuzzy LP with fuzzy parameters to civil engineering problems and the potential advantages of the resultant information.

4. Fuzzy numbers

Fuzzy numbers are defined by fuzzy sets which are convex single-point and normal. Two special classes of fuzzy numbers are used in practice, i.e. triangular and trapezoidal. If $\tilde{A}$ is a fuzzy number then the membership values can be given as

$$\mu_{\tilde{A}}(x) = \mu_{\tilde{A}}(x; a, b, c) = \begin{cases} x - a/b - a & \text{if } a \le x \le b \\ c - x/c - b & \text{if } b \le x \le c \\ 0 & \text{if } x > c \text{ or } x < a \end{cases}$$

If triangular

$$\mu_{\tilde{A}}(x) = \mu_{\tilde{A}}(x; a, b, c, d) = \begin{cases} x - a/b - a & \text{if } a \le x \le b \\ 1 & \text{if } b \le x \le c \\ d - x/d - c & \text{if } c < x \le d \\ 0 & \text{if } x > d \text{ or } x < a \end{cases} \quad \text{If trapezoidal}$$

Triangular fuzzy numbers (TFN) can have equal spread on either side as shown in Fig. 1 for which p is the centroid and c is the spread. The membership values for such fuzzy parameter 'approximately p' with center p and spread/width c is given as

$$\mu_{\tilde{A}}(a) = \min_j [\mu_{A_j}(a_j)] \tag{1}$$

where

$$\mu_{\tilde{A}_i}(a_j) = \begin{cases} 1 - \left(\dfrac{|p_j - a_j|}{c_j} \right), & p_j - c_j \le a_j \le p_i + c_i \text{ and } c_j > 0 \\ 0, & \text{otherwise} \end{cases}$$

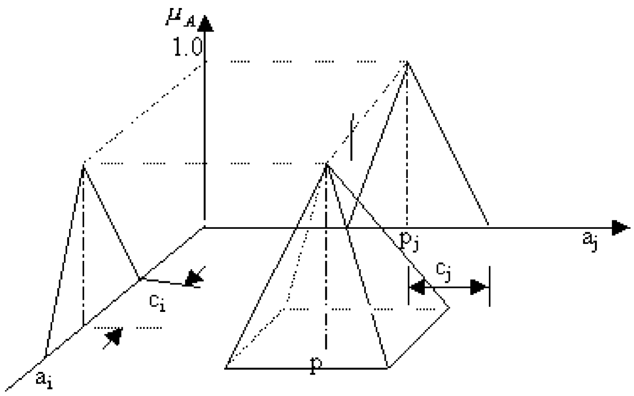

Fig. 1. Fuzzy parameter (approximately p).

or in a vector form $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$ can be represented as $\tilde{A} = \{p, c\}$ where $p = (p_1, p_2, \ldots, p_n)^t$ and $c = (c_1, c_2, \ldots, c_n)^t$

5. Extension principle

The principle of fuzzifying crisp function is called Extension Principle. It is a basic identity that allows extending the domain of a function from crisp points to fuzzy sets in a universe. Let a relation $y = f(x)$ between one independent variable x and one dependent variable y, where f is of analytic form and x, y are deterministic. This relation is a single-input and single-output process, where the transfer function represents the mapping provided by the general function, where

f as $x \rightarrow f(x) \rightarrow y$

But in a typical case if x is a fuzzy variable, and function f may or may not be fuzzy, then the mapping has to be extended.

Let X, Y are two universes and $\tilde{A}$, $\tilde{B}$, are two fuzzy sets in X and Y, respectively, and f be a function from crisp set X to crisp set Y such that $f : X \rightarrow Y$. When f is a one-to-one mapping, then

$$\mu_{\tilde{B}}(y) = \mu_{\tilde{A}}[f^{-1}(y)], \qquad y \in Y$$

If f is not one-to-one then membership value is

$$\mu_{\tilde{B}}(y) = \max_{x \in f^{-1}(y)} \mu_{\tilde{A}}[f^{-1}(y)], \qquad y \in Y \tag{2}$$

where $f^{-1}(y)$ denotes the set of all points $x \in X$ such that $f(x) = y$.

For example, if '$\times$' denote general multiplication, then the multiplication between the two fuzzy numbers $\tilde{A}$, $\tilde{B}$ denoted by $\tilde{A} \times \tilde{B}$ on universe Z, then using the extension principle

$$\mu_{\tilde{A}\times\tilde{B}}(z) = \bigvee_{x \times y = z} (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(y)) \tag{3}$$

where '$\vee$' denotes the supremum of the set. If more than one of the combinations of the input variables X_1, X_2 are mapped to the same variable in the outer space Y; i.e. if the mapping is not one-to-one, then take the maximum membership grade of the combination mappings to the same output variable, which can be shown as

$$\mu_{\tilde{A}}(X_1, X_2) = \max_{Y = f(X_1, X_2)} [\min\{\mu_1(X_1), \mu_2(X_2)\}] \tag{4}$$

Eqs. (3) and (4) develop a procedure for extending crisp domains to fuzzy domains.

6. Fuzzy optimization

The classical LP model is defined as

$$\begin{aligned} &\text{Maximize} && Z = CX \\ &\text{Subject to} && AX \le b \;\; X \ge 0 \end{aligned} \tag{5}$$

Here, $X=\langle x_1,x_2,\dots,x_n\rangle^{\mathrm{T}}$ is a vector of variables, $A=[a_{ij}]$, where $i\in N_m$ and $j\in N_n$ is a constraint matrix, and $b=\langle b_1,b_2,\dots,b_n\rangle^{\mathrm{T}}$ is a right hand side vector. The optimal values for these problems can be achieved by graphical method or with simplex methodology. Eq. (5) is very effective as far as the constraints and their coefficients are crisp, but in many practical situations, the constraints are not crisp and do not have a precise value rendering them to be given some flexibility (Fig. 2). Hence to incorporate these vague factors into the mathematical equations, fuzzy LP is used. The generalized fuzzy LP is shown as

$$\text{Maximize}\quad \sum_{j=1}^{n}\tilde{C}_jX_j \tag{6}$$

$$\text{Subject to}\quad \sum_{j=1}^{n}\tilde{A}_{ij}X_j\lesssim\tilde{B}_i,\quad i\in N_n,\quad X_j\geq 0\ (j\in N_m)$$

where $\tilde{A}_{ij}$, $\tilde{B}_i$ and $\tilde{C}_j$ are fuzzy numbers and X_j are variables whose states are fuzzy numbers ($i\in N_m$, $j\in N_n$). Here $\lesssim$ denotes the ordering of fuzzy numbers or *approximately less than or equal to*. The fuzziness can be in the availability of resources, coefficients of objective functions, coefficients of the constraints, or combination of the three basic types.

6.1. Fuzzy linear programming with fuzzy parameters

Classical LP can be shown as

$$\max_x a^{\mathrm{t}}x=a^{\mathrm{t}}x^*$$

Subject to $\quad Ax\leq b\ \ and\ \ x\geq 0$

The goals are transferred into constraints and the LP model of the problem can be:

$$\begin{array}{ll} a_{01}x_1+a_{02}x_2+\cdots+a_{0n}x_n\geq b_0 & \text{goal}\\ a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n\geq b_1 & \text{Constraint}\\ \vdots & \\ a_{i1}x_1+a_{i2}x_2+\cdots+a_{in}x_n\geq b_i & \text{goal}\\ \vdots & \\ a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n\geq b_m & \text{Constraint}\end{array} \tag{7}$$

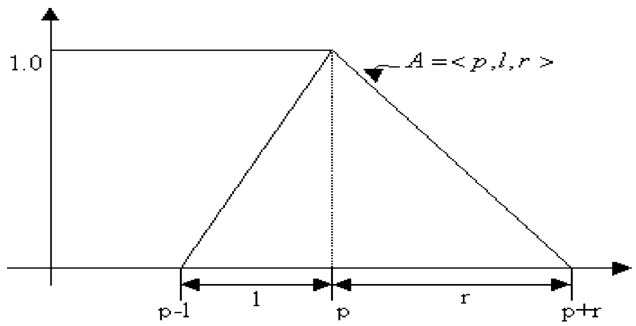

Fig. 2. Fuzzy number with flexibility.

assuming all parameters to be fuzzy in the above equation the problem changes to

$$\begin{array}{l} \tilde{Y}_0=\tilde{B}_0x_0+\tilde{A}_{01}x_1+\tilde{A}_{02}x_2+\cdots+\tilde{A}_{0n}x_n\gtrsim 0\\ \tilde{Y}_1=\tilde{B}_1x_0+\tilde{A}_{11}x_1+\tilde{A}_{12}x_2+\cdots+\tilde{A}_{1n}x_n\gtrsim 0\\ \vdots\\ \tilde{Y}_i=\tilde{B}_ix_0+\tilde{A}_{i1}x_1+\tilde{A}_{i2}x_2+\cdots+\tilde{A}_{in}x_n\gtrsim 0\\ \vdots\\ \tilde{Y}_m=\tilde{B}_mx_0+\tilde{A}_{m1}x_1+\tilde{A}_{m2}x_2+\cdots+\tilde{A}_{mn}x_n\gtrsim 0 \end{array} \tag{8}$$

where $x_0=1$ and $\gtrsim$ shows the fuzzified version of $\geq$ (interpreted as almost positive).

In vector form, Eq. (8) can be written as $\tilde{Y}=\tilde{A}x\geq 0$ where

$$\tilde{A}=\begin{Bmatrix}\tilde{A}_0\\ \tilde{A}_1\\ \vdots\\ \tilde{A}_m\end{Bmatrix}=\begin{Bmatrix}\tilde{B}_0,\tilde{A}_{01},\dots,\tilde{A}_{0n}\\ \tilde{B}_1,\tilde{A}_{11},\dots,\tilde{A}_{1n}\\ \vdots\\ \tilde{B}_m,\tilde{A}_{m1},\dots,\tilde{A}_{mn}\end{Bmatrix} \tag{9}$$

and

$$\tilde{A}_j=(\tilde{B}_j,\tilde{A}_{j1},\dots,\tilde{A}_{jn})=\{p_j,c_j\}$$

where $p_j=(p_{j0},p_{j1},\dots,p_{jn})^{\mathrm{t}}$ and $c_j=(c_{j0},c_{j1},\dots,c_{jn})^{\mathrm{t}}$ are the center and the spread values of the variables considered to be fuzzy in nature. The membership function for which is given by Eq. (10) [11]

$$\mu_{Y_i}(y)=\begin{Bmatrix} 1-\dfrac{\left|y-\sum_{i=1}^{n}p_ix_i\right|}{\sum_{i=1}^{n}c_i|x_i|}, & x_i\neq 0\\ 1, & x_i=0,y=0\\ 0, & x_i0,y\neq 0\end{Bmatrix} \tag{10}$$

$\tilde{Y}_i$ is almost positive and is denoted by $\tilde{Y}_i\gtrsim 0$, i.e.

$$\tilde{Y}_i\gtrsim 0\Leftrightarrow\mu_{Y_i}(y)\leq 1-h,\quad\sum_{i=1}^{n}p_ix_i\geq 0$$

where (h) is the degree by which $\tilde{Y}_i\gtrsim 0$ and the larger the (h) the stronger the meaning of ‘almost positive” and is shown in Fig. 3. If $Y=0$, then the membership value becomes:

$$\mu_{\tilde{Y}_0}(0)=1-\frac{\sum_{i=1}^{n}p_0x_0}{\sum_{i=1}^{n}c_0|x_0|} \tag{11}$$

$$\text{ad}\ \ \mu_{\tilde{Y}_m}(0)=1-\frac{\sum_{i=1}^{n}p_mx_m}{\sum_{i=1}^{n}c_m|x_m|},\quad \text{where}\,x_i>0 \tag{12}$$

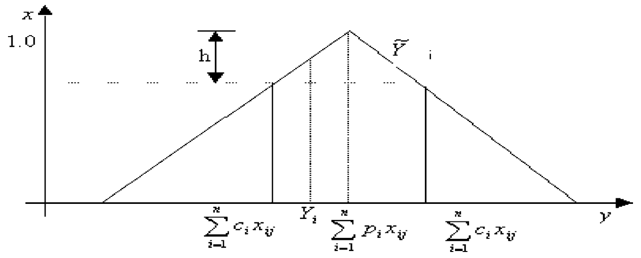

Fig. 3. Explanation of $\tilde{Y}$.

The above Eqs. (11) and (12) can be equated to

$$(p_0 - hc_0)^t x \geq 0 \text{ or} \quad (13)$$

$$(p_m - hc_m)^t x \geq 0$$

Therefore, the problem is reduced to finding the largest degree that is compatible with Eq. (13) and evaluates h and x.

Hence the fuzzy mathematical programming problem with complete fuzziness is

$$\max \; h = h^* \quad (14)$$

$$\text{subject to} \quad (p_j - hc_j)^t x \geq 0 \;\; (j = 0, \ldots n; \; 0 \leq h \leq 1)$$

The solution of x^* ensures that fuzzy inequalities satisfies with a degree of more than h^*.

7. Case study

A case study of Sri Ram Sagar Project constructed across Godavari River at Nizamabad district, Andhra Pradesh, India is considered in this paper. The project was estimated to be around US$ 3.56 million, where 25% was estimated for construction equipment, out of which 15% was contractor's profit. Since the ambiguity exists in the project environment, contractor does not have an exact estimate of the number of pieces of equipment required. The approximate number of pieces of equipment required at the site, approximate cost, approximate availability, rent and the approximate number of days to be hired, etc. are as shown in Table 1, where ~ means approximate, with flexibility as given in the sub column (Fl). The objective is to identify the exact number of equipments to be bought/rented.

7.1. Formulation of the problem

Let x_i be the variable representing the number of pieces of equipment to be bought and y_i are the number of pieces of equipment to be rented, where $i = 1, 2, \ldots, 11$. Here, x_1 represents the number of batching and mixing plants to be bought, and y_1 represents the number of batching and mixing plants to be rented. Here, cost of equipment is fuzzy and appropriate flexibility is incorporated using Eq. (14).

Batching and mixing plant costs ~US$ 100,000 with a flexibility of US$ 14,584. Using Eq. (14), the cost of buying the equipment is expressed as $(100\,000 - 14\,584)x_1$. Since the number of days for renting equipment is fuzzy or rather approximate, the approximate cost incurred for renting each equipment is found by multiplying the fuzzy number of days with the crisp amount of rent of equipment per day. For example, if the number of days of hiring batching and mixing plant equipment is ~60 with a crisp amount of rent per day as US$ 104, then the expected amount without giving flexibility is $\sim 60 \times 104 = \sim 6240$, whereas if a flexibility of 7 days is considered, then the flexible rent can be up to $7 \times 104 = 728$. Hence taking 6240 as centroid and 728 as the spread, which can be expressed using Eq. (14) as $(6240 - 728h)y_1$.

Similarly, other values are found by keeping the budget within the range by incorporating the flexibility of US$ 20,834. The problem is formulated as follows.

Table 1
Details of equipments with expected values (Ev) and flexibilities (Fl)

A	*B*		*C*		*D*	*E*		*F*	
	Ev	Fl	Ev	Fl		Ev	Fl	Ev	Fl
Batching and mixing plant	~3	1	~100,000.00	14,584.00	104.00	~4	1	~6240	728
Transit mixers	~12	2	~14,584.00	2604.00	20.80	~14	2	~1248	208
Compressor	~2	1	~6250.00	1042.00	16.70	~4	1	~1503	251
Rippers	~10	2	~9375.00	1563.00	12.50	~9	2	~375	63
Dozers	~1	0	~10,417.00	3125.00	16.70	~3	1	~1503	167
Excavators	~3	1	~41,667.00	6250.00	83.30	~4	1	~5000	833
Tractors	~4	1	~8334.00	2187.50	62.50	~5	2	~11,250	1250
Crushers	~1	0	~20,834.00	3646.00	62.50	~3	1	~1875	313
Diesel road rollers	~2	0	~8334.00	1042.00	6.25	~3	1	~750	125
Pavers	~3	1	~10,417.00	2292.00	20.80	~4	1	~1248	208
Tankers	~5	1	~4167.00	1146.00	10.40	~5	1	~1877	156

A: Type of equipment; *B* : minimum number required; *C* : cost of each equipment ($); *D* : Rent of each equipment per day ($); *E* : number of equipments that can be hired; *F* : duration of service (days); Ev: expected numbers; Fl: flexibility.

The objective function

$$(356000-20834h)-((100000-14584h)x_1 +(6240-728h)y_1+(14584-2604h)x_2+(1248-208h)y_2 +(6250-1042h)x_3+(1503-251h)y_3+(9375-1563h)x_4 +(375-63h)y_4+(10417-3125h)x_5+(1503-167h)y_5 +(41667-6250h)x_6+(5000-833h)y_6 +(8334-2187.5h)x_7+(11250-1250h)y_7 +(20834-3646h)x_8+(1875-313h)y_8 +(8334-1042h)x_9+(750-125h)y_9+(10417-2292h)x_{10} +(1248-208h)y_{10}+(4167-1446h)x_{11} +(1872-156h)y_{11})\geq 0$$

7.2. Equipment constraints

From Table 1, the number of batching and mixing plants are around 3 with a flexibility of 1 and the number to be hired should not be below 4 with a flexibility of 1. Incorporating these values in Eq. (14), the following equations can be found

$$(-3-h)+(x_1-h)+(y_1-h)\geq 0 \text{ and } (4-h)-(y_1-h)\geq 0$$

Therefore, the complete mathematical formulation after incorporating the flexibilities, without any distinction between the goals and the constraints with all integer values except h is:

Maximize h

Subject to

$$(356000-20834h)-((100000-14584h)x_1 +(6240-728h)y_1+(14584-2604h)x_2+(1248-208h)y_2 +(6250-1042h)x_3+(1503-251h)y_3+(9375-1563h)x_4 +(375-63h)y_4+(10417-3125h)x_5+(1503-167h)y_5 +(416\text{and}67-6250h)x_6+(5000-833h)y_6 +(8334-2187.5h)x_7+(11250-1250h)y_7 +(20834-3646h)x_8+(1875-313h)y_8+(8334-1042h)x_9 +(750-125h)y_9+(10417-2292h)x_{10}+(1248-208h)y_{10} +(4167-1446h)x_{11}+(1872-156h)y_{11})\geq 0$$

$$(-3-h)+(x_1-h)+(y_1-h)\geq 0 \text{ and } (4-h)-(y_1-h)\geq 0$$

$$(-12-2h)+(x_2-h)+(y_2-h)\geq 0 \text{ and } (14-2h)-(y_2-h)\geq 0$$

$$(-2-h)+(x_3-h)+(y_3-h)\geq 0 \text{ and } (4-h)-(y_3-h)\geq 0$$

$$(-10-2h)+(x_4-h)+(y_4-h)\geq 0 \text{ and } (9-2h)-(y_4-h)\geq 0$$

Table 2

The number of equipments brought (X)/rented (Y) as calculated

Variable\equipments	1	2	3	4	5	6	7	8	9	10	11
Bought (X)	1	2	0	5	0	1	6	0	0	1	3
Rented (Y)	4	13	4	8	3	4	0	3	3	4	4

$$-1+(x_5-h)+(y_5-h)\geq 0 \text{ and } (3-h)-(y_5-h)\geq 0$$

$$(-3-h)+(x_6-h)+(y_6-h)\geq 0 \text{ and } (4-h)-(y_6-h)\geq 0$$

$$(-4-h)+(x_7-h)+(y_7-h)\geq 0 \text{ and } (5-2h)-(y_7-h)\geq 0$$

$$-1+(x_8-h)+(y_8-h)\geq 0 \text{ and } (3-h)-(y_8-h)\geq 0$$

$$-2+x_9+(y_9-h)\geq 0 \text{ and } (3-h)-(y_9-h)\geq 0$$

$$(-3-h)+(x_{10}-h)+(y_{10}-h)\geq 0 \text{ and } (4-h)-(y_{10}-h)\geq 0$$

$$(-5-h)+(x_{11}-h)+(y_{11}-h)\geq 0 \text{ and } (5-h)-(y_{11}-h)\geq 0$$

Solving the above non-linear inequalities, using LINGO6, the value of h is found to be 0.67. The final results are tabulated in Table 2. With fuzzy data, the optimal solution is arrived with a satisfaction criterion of 0.67. This means that all the constraints are satisfied with a satisfaction value of more than 0.67. The optimal value for the budget constraint is US$ 363767.13, and is in the specified range. This is as shown in Fig. 4.

8. Discussions

Several important observations are made from the above analysis. The objective value increases with the increase in the width of the TFN. Even though this case study considers only TFN, other types, such as trapezoidal fuzzy numbers can also be incorporated. The decrease in the number of pieces of equipment to be bought and an increase in the number of equipment to be rented will surely decrease the value of objective function. Because of the market

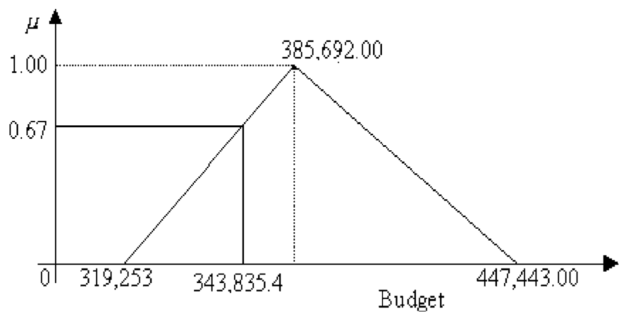

Fig. 4. Fuzzy set of budget constraint with $h = 0.67$.

conditions they are not reduced. The equipments once bought can be transferred to other sites whenever the need arises in the project environment.

9. Conclusions

Construction operations involve many uncertain variables which are vague, qualitative, and approximately defined in the project environment. With the advent of fuzzy logic, incorporation of these uncertain variables into the decision analysis has become much simpler. This paper addressed the application of fuzzy LP with fuzzy constraints that incorporates an efficient computational technique for equipment deployment and a more suitable model for modeling approximate numbers. Compared with traditional LP and fuzzy LP models, this method allows incorporating complete fuzziness in the problem.

The proposed methodology for the optimal deployment of construction equipment is considered effective and practical, since it considers approximate numbers, which involve both technical and economical aspects for obtaining optimal numbers. This methodology can be implemented for the planning and design and in construction phases of the project. The implementation of the methodology in the planning and design phases can determine the exact number of equipment to be deployed. On the other hand, the implementation of the methodology in the construction phase can assist to achieve exact figures in every construction activity. The results indicate that the equipments are not only optimally deployed but also the uncertainty can be handled successfully. Although this model considers only equipments to be bought and rented, for the sake of high precision, other cases such as transfer of equipments from other sites can also be incorporated.

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References

[1] Arditi D, Kale S, Tangkar M. Innovation in construction equipment and its flow into the construction industry. J Construct Engng Mgmt, ASCE 1998;123(4):371–8.
[2] Leu S, Chen A, Yang C. Fuzzy optimal model for resource-constrained construction scheduling. J Construct Engng Mgmt, ASCE 1999;13(3):207–16.
[3] Bellman RE, Zadeh LA. Decision making in fuzzy environment. Mgmt Sci 1970;17(4):141–54.
[4] Zadeh LA. Outline of a new approach to the analysis of complex systems and decision processes. IEEE Trans Syst Man Cybern 1973; 3(1):28–44.
[5] Sasikumar K, Mujumdar PP. Fuzzy optimization model for water quality management of a river system. J Water Resour Plan Mgmt 1996;124(2):79–88.
[6] Ayyub BM, Haldar A. Project scheduling using fuzzy set concepts. J Construct Engng Mgmt, ASCE 1984;110(2):189–204.
[7] Wang SQ, Tiong RLK, Ting SK, Chew D, Ashley D. Evaluation and competitive tendering of BOT power plant project in China. J Construct Engng Mgmt, ASCE 1998;124(4):333–47.
[8] Kumar VSS, Hanna AS, Adams T. Assessment of working capital requirements by fuzzy set theory. J Engng Construct Architect Mgmt 2000;7(1):93–107.
[9] Skibniewski M, Armijos A. Linear programming approach to construction equipment and labor assignments. Civil Engng Syst 1990;7(1):44–50.
[10] Mohan S, Jothi Prakash V. Fuzzy system modelling for optimal crop planning. Inst Engrs 2000;81:9–17.
[11] Tanaka H, Asai K. Fuzzy linear programming problems with fuzzy numbers. Fuzzy Sets Syst 1984;13:11–23.
[12] Cross V, Cabello M. Fuzzy interactive multi objective optimisation on borrowing/lending problems. IEEE Proc ISUMA-NAFIPS 1995; 513–8.
[13] Lorterapong P, Moselhi O. Project-network analysis using fuzzy sets theory. J Construct Engng Mgmt, ASCE 1996;122(4):308–18.