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About Shewhart control charts to monitor the Weibull mean based on a Gamma distribution

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Abstract

A control chart for the mean of a process whose quality characteristic follows a Weibull distribution is evaluated. In this sense, a transformation that follows an exponential distribution is used involving the parameters of scale and form of a Weibull distribution. Such a transformation allows to obtain analytically the control limits to monitor the mean using the Gamma distribution. Compared with recent results in the literature, lower average run lengths after a change (ARL_1) values are measured without the need to use intensive Monte Carlo simulation to obtain the control limits as well as to calculate the ARL_1 values. A numerical example is presented in detail to illustrate the efficiency and effectiveness of the developed control chart.

KEYWORDS

average run length, control charts, exponential distribution, Gamma distribution, variable transformations, Weibull distribution

1 | INTRODUCTION

The monitoring of stable processes aims to detect possible increases in the process dispersion and/or changes in the mean with respect to a target value in order to verify the stability of the process. Control charts are recognized as a very useful tool in process monitoring. These charts compare sample results with previously calculated threshold called control limits. Usually, a point outside the control limits indicates an out-of-control state signaling the presence of special causes in the process. Otherwise, the state under control is assigned to a process exempt from special causes. Parameters such as mean, variance, nonconforming fraction, among others, can be monitored through control charts.

Although initially control charts were applied to manufacturing processes in engineering, it is currently common to witness their use in areas such as health care,[1,2] health surveillance service,[3,4] social network,[5,6] financial surveillance,[7,8] education organizations,[9–11] among others. The theory and method of statistical process control focuses on ease of use and interpretation for end users and the use of control charts helps to understand the behavior of processes or systems over time. Recently, control charts have been used, for example, for count and growth or exponential decline data for use in a pandemic. Especially in studies that analyzed data related to COVID-19 cases in certain regions of the world, control charts showed cases reported daily. Hybrid control chart methods were developed as presented by Perla et al.[12] and Parry et al.[13] In Perla et al.,[12] for example, control charts were used to visualize times and phases of the pandemic. Other interesting studies related to the pandemic caused by COVID-19 can be viewed at Inkelas et al.,[14] Bergaman et al.,[15] and Shah et al.[16]

In most applications, it is usual for the normal distribution to be indicated to describe the behavior of the quality characteristic, which is adequate and facilitates the analysis as in Abbas,[17] Abu-Shawiesh et al.,[18] Khoo and Ariffin,[19] Tailor,[20,21] and others. Additionally, in situations where the quality characteristic is not normally distributed, it is common to propose an adequate transformation for the data, as in Chou[22] and in Fernandes.[23] Some cases of interest such as, for example, survival time, strength, tension, among others, characterize a non-normality in their distribution. However, studies based on the exact distribution of the characteristic of interest have many advantages. Abernethy[24] and Rinne[25] described that the Weibull distribution proves to be quite suitable for these situations because it is more flexible as it can have different forms. This distribution is then an alternative to the normal distribution for such asymmetric situations.

Some contributions in terms of construction of control charts in processes adjusted by the Weibull distribution are presented in the literature, for example, in Nelson,[26] a chart of the median and amplitude was proposed to monitor the parameters of a Weibull process. Ramalhoto and Morais[27] proposed Shewhart control charts for the scale parameter with fixed or variable sampling intervals. Pascual[28] suggested a control chart for the mean employing an exponentially weighted moving average (EWMA) based on the sampling amplitude of the logarithms of the data. Pascual and Zhang[29] presented a control chart to monitor the shape parameter based on the amplitude of random samples from the minimum value distribution. Chein[30] used a Shewhart control scheme based on two control charts to simultaneously monitor the shape and scale parameters of Weibull data without subgrouping. Already Dickinson[31] makes use of a CUSUM plot to monitor mean lifetime with censored data from a Weibull distribution. Faraz et al.[32] proposed Shewhart charts to monitor the scale and shape parameters after transforming a controlled Weibull distribution into a standard normal distribution using an error function. More recently, a study focused on monitoring the shape parameter of a Weibull renewal process was suggested by Zhang et al.[33] Based on a new statistic with an approximately normal distribution, this study analyzes the performance of a new Shewhart-type control chart, called Beta Chart. Arif and Aslam[34] presented a control chart for Weibull using a weighted moving average statistic that is based on the minimum and maximum statistic. Aiming to obtain an improvement in the monitoring of the mean of a process with a random variable following a Weibull distribution, Ho et al.[35] proposed the inclusion of supplementary rules in the traditional Shewhart control chart. Other interesting studies can be seen in Aslam et al.,[36] Gong et al.,[37] Khan et al.,[38] and Huwang et al.[39]

Recently, Fernandes et al.[23] proposed an $\bar{X}$ control chart to monitor the mean of a Weibull process, assuming that the shape parameter does not change. The proposed monitoring was performed without the need for data transformation. However, the control limits as well as the average run length (ARL) values were obtained using Monte Carlo simulation. In this article, an alternative is proposed to the charts from Fernandes et al.[23] to control the quality characteristic X with Weibull distribution through the transformed variable $Y = (\frac{X}{\gamma})^{\delta}$, in which γ is the scale parameter and δ, the shape parameter. As Y follows an exponential distribution, Exp(1), the sum of n independent and identically distributed random variables will have a Gamma distribution with parameters n and 1, respectively, shape and scale. This fact will allow one to obtain the exact control limits and also the calculation of the ARL value. Therefore, in relation to the chart from Fernandes et al.,[23] there will be an advantage of not using Monte Carlo simulation, in addition to obtaining lower values of ARL_1 (with ARL_0 fixed at 370.4) for most displacements on the mean.

The remaining of this article is organized as follows. The Weibull distribution is presented in Section 2. Control charts to monitor $\bar{X}$ and $\bar{Y}$ observations of the Weibull process are described in Section 3. Comparisons among the $\bar{X}$ and $\bar{Y}$ control charts are seen in Section 4. Application to a real data set is presented in Section 5. Final remarks are outlined in Section 6.

2 | WEIBULL DISTRIBUTION AND SOME PROPERTIES

Let X be a quality characteristic following a Weibull distribution, $X \sim \text{Weibull}(\delta, \gamma)$, in which $\delta > 0$ (shape parameter) and $\gamma > 0$ (scale parameter). The probability density function (PDF) of X is given as follows:

$$f(x|\delta,\gamma) = \frac{\delta}{\gamma}\left(\frac{x}{\gamma}\right)^{\delta-1} \exp\left[-\left(\frac{x}{\gamma}\right)^{\delta}\right], \ x > 0, \tag{1}$$

in which the mean and variance are given, respectively, by

$$\begin{cases} \mathbb{E}(X) = \mu = \gamma\Gamma(1/\delta + 1) \text{ and} \\ \mathbb{V}(X) = \sigma^2 = \gamma^2\left[\Gamma(2/\delta + 1) - \Gamma^2(1/\delta + 1)\right]. \end{cases} \tag{2}$$

Note that if the shape parameter δ does not change, then the control of the mean is equivalent to control the scale parameter γ. If $Y = (\frac{X}{\gamma})^{\delta}$, then Y follows the exponential distribution $Y \sim \text{Exp}(1)$, as described in Johnson at al.[40] The result can be obtained by noting that the cumulative distribution function (CDF) of Y can be written as $P(Y \leq y) = P(X \leq y^{1/\delta}\gamma)$ and consequently the PDF of Y is the derivative of its CDF, which gives $Y \sim \text{Exp}(1)$. Thus, the Weibull distribution can be characterized as the distribution of a random variable X such that the random variable Y has an exponential distribution with a mean equal to 1. Additionally, for a given sample $(Y_1, Y_2, \ldots, Y_n)$ of independently and identically distributed (*iid*) random variables of Y, then the random variable $Z = \sum_{i=1}^{n} Y_i$ has a Gamma distribution with parameters n and 1. Note that no results (not even approximations) have been known about the sums of Weibull random variables as noted by Nadarajah[41] and according to Fernandes et al.[23] even the Central Limit Theorem assures that X follows asymptotically a normal distribution, although such a convergence is not reliable as it depends on the sample size and μ. Thus the exact distribution of $\bar{X}$, for $X_i \sim \text{Weibull}(\delta, \gamma)$, is unknown.

3 | OBTAINING CONTROL LIMITS FOR $\bar{X}$ AND $\bar{Y}$ CONTROL CHARTS

Let $(X_1, X_2, \ldots, X_n)$ be an *iid* sample of size n from a population $X_i \sim \text{Weibull}(\delta, \gamma)$. When the process is in control, its mean is denoted by μ_0 and, when it is out of control, by $\mu_1 = \mu_0 \times (1 + d)$, in which d is the shift size, which can be positive (increase in the mean) or negative (decrease in the mean). It is assumed that the shape parameter δ does not change, as considered by Ramalhoto and Morais,[27] Pascual and Park,[42] Fernandes et al.,[23] and Ho et al.[43] Pascual and Park[42] observed that there are practical situations when this assumption is valid. The value of δ may be an inherent property of the process, that is, it is specified by mechanical and material properties. Additionally, Fernandes et al.[23] used the empirical distribution function of $Z_0 = \frac{(\bar{X}|\mu_0) - \mu_0}{\sigma_0/\sqrt{n}}$ to build the control charts. Using 20,000 simulated values from a Weibull distribution with mean μ_0 and standard deviation σ_0, Fernandes et al.[23] calculated the upper and lower control limits using the $(\alpha/2)$th and $(1 - \alpha/2)$th quantiles, respectively, that is, $q_{\alpha/2}$ and $q_{1-\alpha/2}$ of Z_0. The selection of α is such that an $\text{ARL}_0 = 1/\alpha = 370.4$ is achieved. The control chart performance is expressed in terms of $\text{ARL}_1 = \frac{1}{1 - P(q_{\alpha/2} < Z_1 < q_{1-\alpha/2})}$, with $Z_1 = \frac{(\bar{X}|\mu_1) - \mu_0}{\sigma_0/\sqrt{n}}$, also using 20,000 simulated values of out-of-control $\bar{X}|\mu_1$.

In this article, the control of the Weibull population mean will be done by means of the transformed variable $Y = (\frac{X}{\gamma})^{\delta}$. Thus, the goal is to monitor the quantity $\bar{Y}$ since shifts in the shape parameter δ or in the scale parameter γ will alter the distribution of $\bar{Y}$ and changes will be signalized in $\bar{X}$. In this sense, let $(Y_1, Y_2, \ldots, Y_n)$ be an *iid* sample from an exponential distribution with mean 1. The upper control limit (UCL) and the lower control limit (LCL) are obtained such that the following expressions are satisfied:

$$P\left(Z = \sum_{i=1}^{n} y_i > n\text{UCL}\right) = \alpha/2 \tag{3}$$

and

$$P\left(Z = \sum_{i=1}^{n} y_i < n\text{LCL}\right) = \alpha/2. \tag{4}$$

As noted in Section 2, $Z = \sum_{i=1}^{n} y_i$ has a Gamma distribution with parameters n and 1, denoted by $\text{Gamma}(n, 1)$, with CDF $F_Z(z)$. Consequently, the UCL and LCL are numerically calculated by the following expressions:

$$\text{UCL} = \frac{F_Z^{-1}\left(1 - \frac{1}{\alpha}\right)}{n} \tag{5}$$

and

$$\text{LCL} = \frac{F_Z^{-1}\left(\frac{1}{\alpha}\right)}{n}. \tag{6}$$

For each sample collected in the production process, $\bar{Y}$ is computed and compared with the UCL and LCL. If $\bar{Y} > \text{UCL}$ or $\bar{Y} < \text{LCL}$, the process will be considered out of control and otherwise, under control.

Although the calculation of Equations (5) and (6) are not numerically simple, they can be easily done with the help of common computational resources. For example, if $n = 5$ and $\alpha = 0.002699796$ (to have an $\text{ARL}_0 \approx 370.4$), then by using the statistical software R,[44] it is found that UCL = qgamma(1−0.0026997962/2,5,1)/5 = 2.878499 and LCL = qgamma(0.002699796/2,5,1)/5 = 0.158372. Even in spreadsheets, for example, such as Excel, the procedure is simple as GAMMA.INV(1−0.002699796/2,5,1)/5 = 2.878499, for the UCL, and GAMMA.INV(0.002699796/2,5,1)/5 = 0.158372, for the LCL.

To compute the ARL_1 values, it must be observed that the parameter of shape δ does not change and considering the average expressed in Equation (2) and $\mu_1 = \mu_0 \times (1 + d)$, it follows that the change in the average is equivalent to changing the scale parameter under control γ_0 to the scale parameter out of control $\gamma_1 = \gamma_0 \times (1 + d)$. The problem in calculating the ARL_1 is that when the process goes out of control, the variable X will have parameters of shape δ and scale γ_1. However, since the transformation $Y = (\frac{X}{\gamma_0})^\delta$ is maintained, the property of Y having an exponential distribution with mean equal to 1 will no longer be valid. Thus, to continue using the properties of the exponential and Gamma distributions, some algebraic manipulations must be performed in such a way that the ARL_1 can be obtained based on $Y = (\frac{X}{\gamma_1})^\delta$. Therefore, the expression of ARL_1 can be given by

$$\text{ARL}_1 = \frac{1}{1-\beta}, \tag{7}$$

in which β may be interpreted as the type 2 error in a hypothesis test. Then, β is obtained from

$$\beta = P\left[n\text{LCL} \le \sum_{i=1}^{n} \left(\frac{X_i|\gamma_1}{\gamma_0}\right)^\delta \le n\text{UCL}\right]. \tag{8}$$

An issue with Equation (8) is that the distribution of $(\frac{X_i|\gamma_1}{\gamma_0})^\delta$ is unknown. However, if the terms inside the square brackets in Equation (8) is multiplied by $(\frac{\gamma_0}{\gamma_1})^\delta$, the following expression is obtained, in which the term $(\frac{X_i|\gamma_1}{\gamma_1})^\delta$ has an exponential distribution with mean 1 and consequently the sum $\sum_{i=1}^{n}(\frac{X_i|\gamma_1}{\gamma_1})^\delta$ will have a Gamma distribution making it possible to calculate the ARL_1:

$$\beta = P\left[n\text{LCL}\left(\frac{\gamma_0}{\gamma_1}\right)^\delta \le \sum_{i=1}^{n} \left(\frac{X_i|\gamma_1}{\gamma_1}\right)^\delta \le n\text{UCL}\left(\frac{\gamma_0}{\gamma_1}\right)^\delta\right]. \tag{9}$$

Note that the calculation of β depends on the ratio $\frac{\gamma_0}{\gamma_1} = (1 + d)^{-1}$, meaning that different values of d result in different values of β.

As in the calculation of the control limits described earlier, the value of ARL_1 can also be calculated using simple computational resources. For example, if $n = 5$, $\delta = 3$, $d = 0.2$, and $\mu_0 = 3.572$, then $\gamma_0 = 4.00$ and $\gamma_1 = \gamma_0 + \gamma_0 \times d = 4.80$. Thus, using R, $\beta =$ pgamma(5*2.878499*(4.00/4.80)^3,5,1)−pgamma (5*0.158372*(4.00/4.80)^3,5,1) = 0.9176019, resulting in an $\text{ARL}_1 = \frac{1}{1-\beta} \cong 12.1362$. In Excel, β is computed as gamma.dist(5*2.878499*(4/4.8)^3,5,1,1)−gamma.dist(5*0.158372*(4/4.8)^3,5,1,) = 0.9176019, which results in an equivalent ARL_1.

4 | NUMERICAL RESULTS

This section consists of three subsections. In Section 4.1, a comparison is made between the ARL_1 of the $\bar{X}$ control chart from Fernandes et al.[23] and the proposed $\bar{Y}$ chart. In Section 4.2, it is suggested to use the control chart $\bar{Y}$ when shifts occur in both parameters (shape and scale). Finally, in Section 4.3, the impact on ARL_0 when both parameters (shape and scale) are unknown is discussed. By a simulation study, it was determined that $m = 300$ random samples of size $n = 5$ are necessary to minimize such impact.

4.1 | Comparing the ARL_1 values of the $\bar{X}$ versus $\bar{Y}$ control charts

In this subsection, a comparison is performed between the $\bar{X}$ control chart proposed by Fernandes et al.,[23] in which the control limits and ARL_1 were obtained by Monte Carlo simulations, with those obtained by the $\bar{Y}$ control chart here proposed, in which the control limits and ARL_1 are obtained exactly using the Gamma distribution.

Table 1 presents the values of ARL obtained from the study considering shifts in two directions, $d \in \{-0.4; -0.3; -0.2; -0.1; -0.05; -0,01; 0; 0.01; 0.05; 0.1; 0.2; 0.3; 0.4\}$, sample size $n \in \{3, 5, 10, 30, 100\}$, and shape parameter $\delta \in \{0.5, 3, 5, 10, 15, 20\}$. The choices of values for d, n, and δ were made in such a way that several practical situations were taken into account. The sample sizes n include typical values used in the statistical process control as observed in Montgomery.[45] The choices for the shape parameter (δ) are those described in many practical cases as in Jiang and Murthy,[46] Rinne,[25] and Abernethy.[24] The selection of d in the range $[-0.4, 0.4]$ has the aim to verify the effective differences between the current proposal and the one presented by Fernandes et al.[23] Shifts in the mean larger than 40% make the two proposals similar in terms of ARL_1. As the shape parameter δ is considered fixed, then the parameter $(1 + d)$ can be interpreted as the change in mean when the process goes out of control, that is, $\mu_1 = \mu_0 \times (1 + d)$ or, equivalently, by changing the scale parameter, that is, $\gamma_1 = \gamma_0 \times (1 + d)$.

Looking at Table 1, it is possible to verify some results. First, it is noteworthy that, in Fernandes et al.'s[23] proposal, it is not easy to calibrate the ARL_0 value to 370.4 since the control limits are obtained by Monte Carlo simulation. In general terms, considering only the cases where $d \neq 0$ (for $d = 0$, the values should be equal to 1, if an $\text{ARL}_0 \approx 370.4$ was adopted), the proposed $\bar{Y}$ control chart yields smaller ARL_1 values in approximately 70% of the cases evaluated. In Figure 1, it is observed that the proposed $\bar{Y}$ chart presented an average percentage of cases with lower ARL_1 higher than the $\bar{X}$ chart. It is noteworthy that for high values of $|d|$, the percentage of ties grows since the two proposals are easily able to detect the change in the average. In Figure 2, the average percent change of the ARL_1 of the $\bar{X}$ and $\bar{Y}$ control charts is observed, defined as $\Delta\%_{\text{ARL}} = \frac{ARL_1\bar{X} - ARL_1\bar{Y}}{ARL_1\bar{X}} \times 100\%$.

Since the average performance of the proposed $\bar{Y}$ control chart was better than the $\bar{X}$ chart and the values of the control limits, as well as the computation related to the ARL_1 values, were calculated analytically, that is, without the use of Monte Carlo simulation (unlike in the $\bar{X}$ chart), the proposed $\bar{Y}$ chart may be considered competitive. In fact, the proposed $\bar{Y}$ control chart can be useful for application in real cases in a simple way, and can even be implemented using common spreadsheets such as Excel that are generally widely available in the business environment.

4.2 | The use of $\bar{Y}$ chart when shifts occur on the scale and shape parameters

In the previous sections, it was considered that the shape parameter (δ) was kept constant as stated in Ramalhoto and Morais,[27] Pascual and Park,[42] Fernandes et al.,[23] and Ho et al.,[43] which allowed up a direct comparison of the current results with those obtained in Fernandes et al.[23] In this section, it will be shown that the control limits expressed in Equations (5) and (6) can also be employed when shifts are observed in both shape and scale parameters (δ and γ). Note that the values of ARL_1 cannot be directly calculated by the expressions (7) and (8) as the shape parameter δ may change. Due to the complexity to get the distribution of $\bar{Y}$ when the process is out of control (when the shape parameter δ shifts), an option was to apply a Monte Carlos simulation procedure (with 50,000 runs) to get the ARL_1 values. To illustrate, let be the in-control parameters as ($\delta_0 = 5; \gamma_0 = 3$), which shifts for $\delta_1 \in \{4; 4.5; 5; 5.5; 6\}$ and $\gamma_1 \in \{2; 2.5; 3.5; 4\}$, as shown in Table 2. The values of (δ_1, γ_1) were chosen in order to contemplate different situations that could be observed in practice (increase and decrease of the parameters under control). Table 2 shows that the control limits here proposed are able to generate effective signaling of shifts at the mean process when changes on scale (γ) and form (δ) parameters occur since $\text{ARL}_1 < \text{ARL}_0$, with $\text{ARL}_0 = 370.4$. The percentage $100 \times d\%$ of the shift in the mean expressed by d, being obtained by $\frac{\mu_1}{\mu_0} - 1$ with $\mu = \gamma\Gamma(1/\delta + 1)$.

4.3 | Effect on ARL when parameters need to be estimated

In many practical situations, past experiences resulting from several samples from the process allow to consider the parameters of scale and form known in the situation under control and the developments described earlier can be used. However, when the scale and shape parameters cannot be considered known, it is necessary to perform the estimation of these

TABLE 1 Values of ARL

		Shift size d																								
		−0.40		−0.30		−0.20		−0.10		−0.05		−0.01		0		0.01		0.05		0.10		0.20		0.30		0.40	
δ	n	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$	$\bar{X}$	$\bar{Y}$
0.5	3	345.71	348.88	390.96	406.57	419.89	432.75	419.22	417.88	397.36	397.08	375.32	376.08	368.51	370.40	356.76	364.59	345.57	340.42	314.66	309.42	265.29	250.51	213.32	200.76	173.25	161.30
	5	245.91	245.94	327.50	326.72	371.77	390.86	405.68	408.28	390.11	395.25	378.24	376.12	377.76	370.40	366.76	364.37	339.95	337.94	311.55	302.16	254.23	233.08	201.84	176.78	157.81	134.77
	10	127.01	126.39	206.56	207.92	289.34	308.37	373.25	383.39	381.21	389.44	373.47	375.99	368.76	370.40	360.05	364.05	345.83	332.80	308.45	286.71	230.01	199.19	168.28	135.66	121.22	94.21
	30	31.05	28.08	72.37	67.41	168.01	153.92	305.25	303.65	364.17	366.00	359.07	375.12	376.36	370.40	356.13	363.11	333.89	314.80	267.52	238.39	168.18	122.84	94.16	65.56	58.62	38.04
	100	4.49	3.65	13.50	11.22	49.71	41.55	165.67	166.23	297.27	300.01	384.02	371.90	374.81	370.40	357.14	360.04	302.48	264.18	189.97	145.32	71.49	45.09	30.86	17.81	14.92	8.70
3	3	14.11	13.04	40.92	40.25	116.66	115.39	305.46	300.33	413.87	420.16	394.32	401.11	372.13	370.40	329.49	333.30	191.62	183.75	88.33	78.93	23.52	19.81	9.35	7.73	4.88	4.10
	5	3.81	3.27	13.44	11.80	51.67	47.95	201.93	192.99	354.89	352.71	401.32	398.40	370.75	370.40	332.34	329.84	170.71	158.34	68.21	57.13	15.01	12.14	5.65	4.59	3.00	2.51
	10	1.20	1.11	2.93	2.43	13.99	11.69	94.11	85.78	249.13	242.24	381.94	390.26	368.24	370.40	326.12	322.57	131.78	116.81	39.49	32.00	7.20	5.65	2.74	2.24	1.62	1.42
	30	1.00	1.00	1.01	1.00	1.86	1.57	18.34	15.14	104.82	90.75	349.04	358.55	377.22	370.40	307.13	297.51	65.42	52.17	12.44	9.22	2.05	1.69	1.14	1.08	1.02	1.01
	100	1.00	1.00	1.00	1.00	1.00	1.00	2.49	2.00	21.78	17.48	313.87	275.90	417.71	370.40	240.27	232.55	16.96	13.06	2.51	2.00	1.02	1.01	1.00	1.00	1.00	1.00
5	3	2.41	1.95	8.91	7.48	38.80	35.84	177.36	169.95	354.46	348.03	404.63	416.36	370.64	370.40	309.50	306.56	116.39	101.69	33.83	27.69	6.71	5.29	2.76	2.27	1.71	1.49
	5	1.10	1.03	2.57	1.97	13.18	10.21	95.63	82.98	257.04	244.92	412.44	407.70	367.86	370.40	303.36	298.81	92.59	77.06	23.46	17.45	4.21	3.17	1.86	1.53	1.28	1.15
	10	1.00	1.00	1.07	1.01	2.94	2.14	31.99	24.59	145.21	125.54	389.20	384.94	372.49	370.40	292.19	282.35	59.47	46.11	11.65	8.28	2.14	1.67	1.20	1.09	1.03	1.01
	30	1.00	1.00	1.00	1.00	1.01	1.00	4.40	2.98	39.11	27.76	330.25	310.45	369.41	370.40	247.22	231.57	20.65	14.48	3.16	2.27	1.07	1.02	1.00	1.00	1.00	1.00
	100	1.00	1.00	1.00	1.00	1.00	1.00	1.07	1.01	5.43	3.61	197.39	176.22	364.70	370.40	164.53	137.15	4.43	2.98	1.12	1.03	1.00	1.00	1.00	1.00	1.00	1.00
10	3	1.00	1.00	1.12	1.02	4.60	3.17	52.22	41.98	191.31	176.51	435.54	433.11	373.83	370.40	257.02	239.63	33.30	26.18	6.25	4.73	1.57	1.35	1.10	1.05	1.02	1.01
	5	1.00	1.00	1.00	1.00	1.54	1.16	18.64	12.44	109.49	87.63	395.88	398.15	378.33	370.40	240.56	220.52	23.14	16.42	4.00	2.86	1.22	1.09	1.02	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	4.29	2.55	39.38	26.56	346.86	325.72	371.31	370.40	214.97	184.28	11.96	7.75	2.11	1.55	1.02	1.00	1.00	1.00	1.00	1.00
	30	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.00	5.59	3.23	212.99	176.72	378.79	370.40	136.37	107.57	3.36	2.15	1.07	1.01	1.00	1.00	1.00	1.00	1.00	1.00
	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.15	1.02	76.39	53.19	361.53	370.40	51.45	36.66	1.14	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
15	3	1.00	1.00	1.00	1.00	1.30	1.07	17.24	11.69	108.27	88.26	428.93	421.61	375.21	370.40	193.15	180.65	12.39	9.36	2.44	1.93	1.08	1.04	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	4.98	2.93	48.47	33.11	376.08	355.91	371.31	370.40	178.02	155.03	8.16	5.55	1.71	1.36	1.01	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.41	1.08	13.33	7.33	280.39	246.89	367.43	370.40	139.83	113.54	4.10	2.65	1.16	1.05	1.00	1.00	1.00	1.00	1.00	1.00
	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.79	1.22	124.10	94.32	373.69	370.40	73.27	49.99	1.42	1.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	32.88	18.55	357.14	370.40	21.53	12.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
20	3	1.00	1.00	1.00	1.00	1.00	1.00	6.34	3.94	60.10	45.07	397.05	391.23	369.60	370.40	150.92	133.82	5.97	4.46	1.50	1.28	1.01	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.97	1.29	22.95	13.61	329.79	302.08	371.22	370.40	128.01	107.23	3.88	2.71	1.19	1.07	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.00	5.32	2.77	220.10	179.79	368.76	370.40	94.90	69.95	2.09	1.49	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.11	1.01	76.76	51.58	361.79	370.40	39.58	25.07	1.07	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	15.72	7.72	384.02	370.40	9.83	5.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

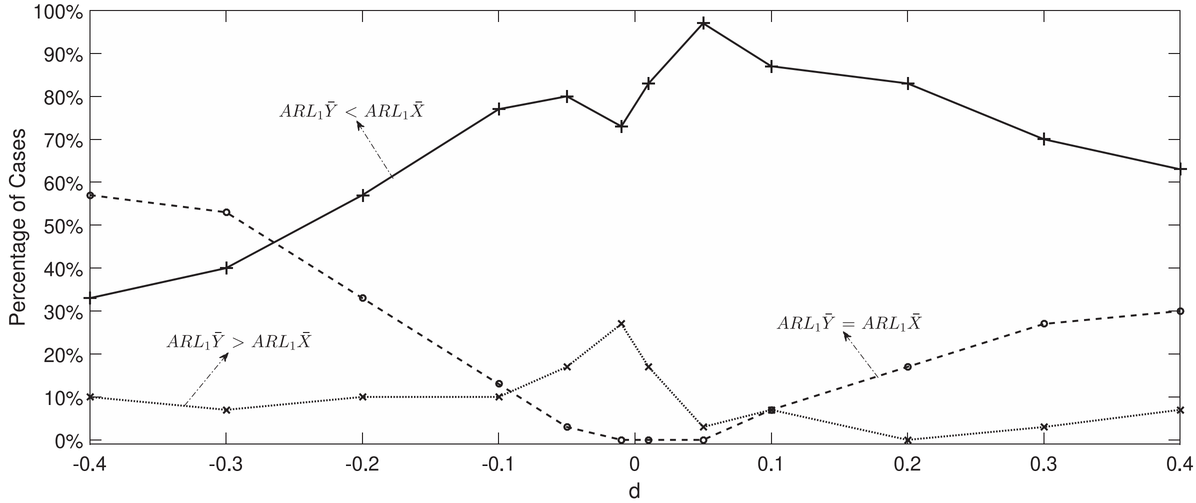

FIGURE 1 Comparisons between the ARL_1 of the $\bar{X}$ and $\bar{Y}$ control charts

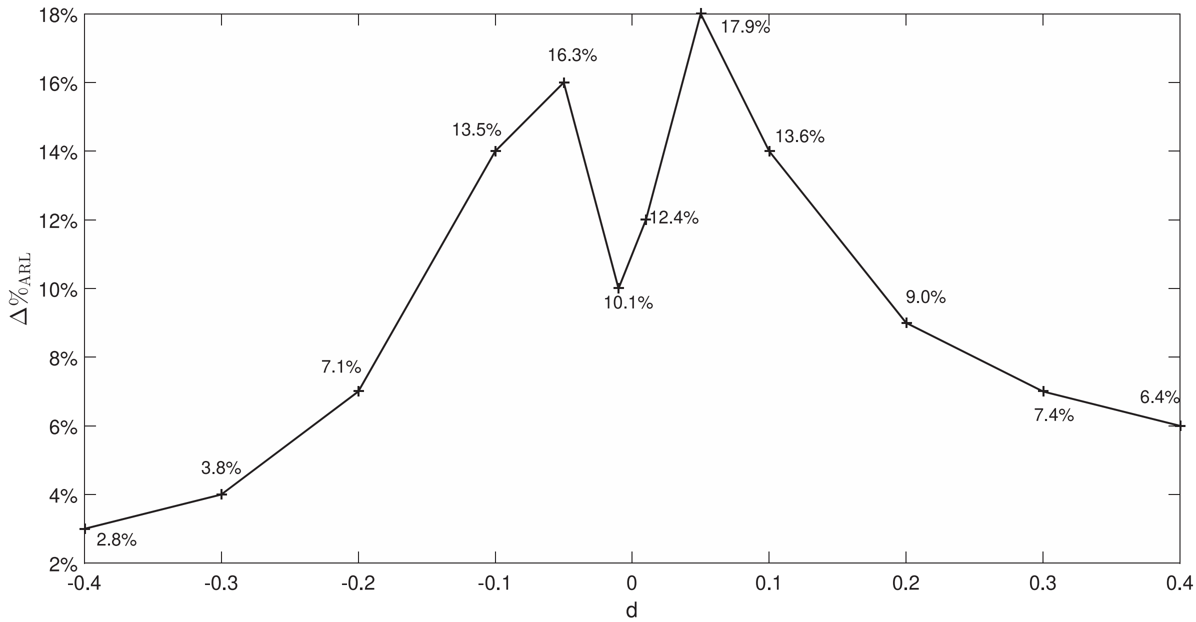

FIGURE 2 Average percent change of the ARL_1 of the $\bar{X}$ and $\bar{Y}$ control charts, $\Delta\%_{\text{ARL}}$

parameters, a step known as Phase I in quality control. In general, m random samples of size n are obtained (assuming the process is in control) and then the scale and shape parameters are estimated. In this article, the estimation will be performed based on the sample size $n \times m$ using maximum likelihood estimators $(\hat{\gamma}; \hat{\delta})$. Once the estimates of the scale and shape parameters are defined, then the control limits are calculated for later use with data from the process in the step called Phase II. Note that even when the shape parameter does not change, it may be unknown.

The problem is that if $n \times m$ is not large enough, then the estimates of the scale and shape parameters can present a large error and consequently the planned ARL_0 can be very different from the desired value (say, around 370.4, for an $\alpha = 0.002699796$), implying in mistaken evaluations of possible values of ARL_1. Essentially, when the parameters of scale γ and shape δ are unknown, they need to be estimated from an initial data set (typically from m subgroups each of size n) and the resulting estimates are used in the control limits. The effects of estimating the limits of the control charts are important on their properties. Most importantly, the run-length distribution is no longer geometric, since the signaling events are no longer independent. Thus, it is no longer true that the ARL_0 of the chart with estimated control limits is equal to $\frac{1}{\alpha}$. This has important implications, since as noted earlier, practitioners sometimes wish to use the results in the

TABLE 2 ARL_1 values for δ_1 and γ_1

			Sample size n				
δ_1	γ_1	d	5	10	15	20	50
5	**3**	**0**	**370.40**	**370.40**	**370.40**	**370.40**	**370.40**
6	4	0.35	1.33	1.03	1.00	1.00	1.00
	3.5	0.18	7.40	2.94	1.83	1.40	1.01
	2.5	−0.16	31.43	3.90	1.61	1.15	1.00
	2	−0.33	1.27	1.00	1.00	1.00	1.00
5.5	4	0.34	1.34	1.04	1.00	1.00	1.00
	3.5	0.17	5.99	2.63	1.73	1.36	1.01
	2.5	−0.16	24.84	3.97	1.74	1.22	1.00
	2	0.33	1.32	1.00	1.00	1.00	1.00
5	4	0.33	1.35	1.04	1.01	1.00	1.00
	3.5	0.17	4.84	2.35	1.62	1.32	1.01
	2.5	−0.17	19.96	4.10	1.92	1.33	1.00
	2	−0.33	1.40	1.00	1.00	1.00	1.00
4.5	4	0.33	1.35	1.05	1.01	1.00	1.00
	3.5	0.16	3.94	2.09	1.51	1.26	1.01
	2.5	−0.17	16.37	4.31	2.18	1.50	1.00
	2	−0.34	1.49	1.00	1.00	1.00	1.00
4	4	0.32	1.35	1.06	1.01	1.00	1.00
	3.5	0.15	3.24	1.85	1.41	1.21	1.00
	2.5	−0.18	13.69	4.65	2.58	1.80	1.03
	2	−0.34	1.61	1.02	1.00	1.00	1.00

standard known case (at least as an approximation) to design control charts even when some of the underlying parameters are unknown.

The exact calculation of the ARL_0 when using estimates $(\hat{\gamma};\hat{\delta})$ is not simple since the joint distribution for $(\hat{\gamma};\hat{\delta})$ is unknown. The use of a multivariate normal approximation was not satisfactory (results not shown). Thus, Monte Carlo simulations will be used by generating $j, (j = 10,000)$ *iid* Weibull$(\gamma;\delta)$ samples of size $n \times m$ and, for each one, the scale γ and shape δ parameters are estimated by the maximum likelihood method, composing the estimates $(\hat{\gamma}_j;\hat{\delta}_j)$. For each set of estimated parameters, k $(k = 100,000)$ *iid* Weibull$(\hat{\gamma}_j;\hat{\delta}_j)$ samples are generated. Thus, for the jth simulation with k samples of size n, the parameter $\hat{\alpha}_j$ of a geometric distribution is estimated, in which $\hat{\alpha}_j$ is given by the percentage of times that $\bar{Y}_k$ is greater than the UCL or lower than the LCL (obtained from (5) and (6), Section 3). Then, for each $\hat{\alpha}_j$, z $(z = 10,000)$ *iid* samples from a geometric distribution Geom$(\hat{\alpha}_j)$ are generated. Finally, all generated values from the geometric distribution are merged into a single vector of size $j \times z$ whose average will be an approximation of the real ARL_0, when the parameters $(\gamma;\delta)$ needed to be estimated. The standard deviation of the run length ($SDRL_0$) is calculated as the average of standard deviations obtained for each of z $(z = 10,000)$ *iid* samples from a geometric distribution Geom$(\hat{\alpha}_j)$. Tables 3 and 4 present ARL_0 and $SDRL_0$ results for $n \in \{5; 10\}$, $m \in \{25; 50; 75; 100; 300\}$, and scale and shape parameters such that averages equal to 3 or 5 are implied. It is observed that, for small values of m, the ARL_0 and $SDRL_0$ can be underestimated or overestimated. However, a value of m around 300 can satisfactorily minimize the problem once the values of ARL_0 and $SDRL_0$ are closer to 370.4. Note that we are evaluating the impact in ARL_0 when estimated are used for δ and γ so the value of d is equal to zero in all cases shown in Tables 3 and 4.

5 | NUMERICAL EXAMPLE

In this section, the numerical example used in previous studies (see Nelson,[26] Ramalhoto and Morais,[27] Pascual and Zhang,[29] and Faraz et al.[32]) is reanalyzed considering that it is wanted to verify whether or not the mean has changed. It is a data set related to the breaking strengths (in gigapascals) of carbon fibers used in manufacturing fibrous composite

TABLE 3 ARL_0 values for estimated parameters

$m \rightarrow$			25		50		75		100		300	
$n \rightarrow$			5	10	5	10	5	10	5	10	5	10
μ_0	δ	γ	ARL_0									
3	0.5	1.500	392.97	348.68	386.40	356.25	379.80	362.03	379.84	368.04	372.20	371.27
	3	3.360	394.18	354.89	381.49	356.76	380.43	359.64	379.49	367.69	371.09	370.33
	5	3.267	395.61	352.00	383.19	362.15	377.70	365.70	374.24	368.79	369.92	370.77
	10	3.153	394.23	353.94	380.95	364.32	378.55	365.53	373.01	367.65	372.13	371.09
	15	3.107	383.30	355.93	383.00	361.52	380.31	363.04	375.48	366.81	373.59	369.88
	20	3.082	394.62	350.65	388.22	357.45	381.26	362.20	380.33	362.76	374.77	370.28
5	0.5	2.500	393.84	351.75	386.07	359.14	382.91	359.35	375.87	366.22	372.88	370.09
	3	5.599	400.04	345.97	381.77	348.72	381.12	364.59	376.75	367.87	372.97	371.18
	5	5.446	397.67	348.89	386.10	354.65	379.14	361.55	374.87	363.52	371.62	369.19
	10	5.256	389.82	347.27	379.96	364.12	375.23	366.58	375.36	367.83	369.89	369.37
	15	5.178	395.05	342.88	386.08	352.87	376.64	364.29	374.70	366.55	374.67	369.97
	20	5.136	396.53	349.41	390.37	362.61	375.47	364.20	373.02	366.56	371.12	370.15

TABLE 4 $SDRL_0$ values for estimated parameters

$m \rightarrow$			25		50		75		100		300	
$n \rightarrow$			5	10	5	10	5	10	5	10	5	10
μ_0	δ	γ	ARL_0									
3	0.5	1.500	388.66	344.62	382.16	352.53	376.76	358.42	377.42	365.58	371.92	370.93
	3	3.360	389.74	350.56	376.80	352.93	377.16	356.33	376.74	365.46	370.80	370.01
	5	3.267	391.38	348.47	379.11	357.75	374.63	362.47	371.47	366.43	369.72	370.40
	10	3.153	388.93	349.25	376.68	360.35	375.47	362.04	370.78	365.32	371.82	370.78
	15	3.107	378.34	351.23	378.93	357.52	376.95	360.19	373.13	364.21	373.27	369.58
	20	3.082	389.48	345.92	383.62	353.13	377.95	359.06	378.08	360.26	374.40	369.93
5	0.5	2.500	388.41	347.19	381.76	355.10	379.42	356.48	373.14	363.67	372.61	369.86
	3	5.599	395.37	340.89	377.09	344.89	377.71	360.99	374.05	365.46	372.69	370.88
	5	5.446	392.51	344.63	381.36	350.70	375.88	358.66	372.80	361.33	371.40	369.00
	10	5.256	385.16	342.33	375.85	359.71	371.85	363.38	372.63	365.27	369.55	369.10
	15	5.178	389.72	339.31	381.27	349.26	372.98	360.90	372.74	364.00	374.30	369.61
	20	5.136	391.54	345.76	386.32	358.61	372.02	361.43	371.10	363.98	370.92	369.92

materials. They are derived from a study done by the U.S. Army Materials Technology Laboratory in Watertown, MA. All these articles reported that the parameters δ and γ were assumed to be both known and equal, respectively, to 4.8 and 3.2 resulting in a mean $\mu_0 = 2.9312$.

Table 5 reproduces the data set. Here, it is assumed that there is an interest to verify if the process mean is changing. Using Equations (5) and (6), it is easy to compute $\text{UCL} = 2.88$ and $\text{LCL} = 0.16$ and to compare them with the values of $\bar{Y}$, calculated for each sample. The samples #13, #14, and #19 are out of control. The graphical representation is similar to the usual $\bar{X}$ control chart based on the normal distribution and can be seen in Figure 3 in which samples #13, #14, and #19 are out of control.

6 FINAL REMARKS

In this article, a new approach to controlling the process mean of an Weibull distribution is presented. It is an analytical alternative to the method proposed by Fernandes et al.,[23] in which intensive Monte Carlo simulations are required at all stages. Based on several situations presented in Table 1, it is concluded that the approach proposed here is competitive

TABLE 5 Data set for the numerical example and decision

n	x_1	x_2	x_3	x_4	x_5	$\bar{y} = \sum_{i=1}^{n} \frac{(\frac{x_i}{\gamma})^\delta}{n}$	Decision
1	3.7	2.74	2.73	2.5	3.6	1.002918607	In control
2	3.11	3.27	2.87	1.47	3.11	0.69409768	In control
3	4.42	2.41	3.19	3.22	1.69	1.40633147	In control
4	3.28	3.09	1.87	3.15	4.9	2.141017515	In control
5	3.75	2.43	2.95	2.97	3.39	1.020530276	In control
6	2.96	2.53	2.67	2.93	3.22	0.62325751	In control
7	3.39	2.81	4.2	3.33	2.55	1.418096902	In control
8	3.31	3.31	2.85	2.56	3.56	0.987312477	In control
9	3.15	2.35	2.55	2.59	2.38	0.418891185	In control
10	2.81	2.77	2.17	2.83	1.92	0.36633555	In control
11	1.41	3.68	2.97	1.36	0.98	0.538880478	In control
12	2.76	4.91	3.68	1.84	1.59	2.071872213	In control
13	3.19	1.57	0.81	5.56	1.73	3.05005027	Out of control
14	1.59	2	1.22	1.12	1.71	0.041047879	Out of control
15	2.17	1.17	5.08	2.48	1.18	1.931567704	In control
16	3.51	2.17	1.69	1.25	4.38	1.256634652	In control
17	1.84	0.39	3.68	2.48	0.85	0.464418608	In control
18	1.61	2.79	4.7	2.03	1.8	1.411946462	In control
19	1.57	1.08	2.03	1.61	2.12	0.065262542	Out of control
20	1.89	2.88	2.82	2.05	3.65	0.645304568	In control

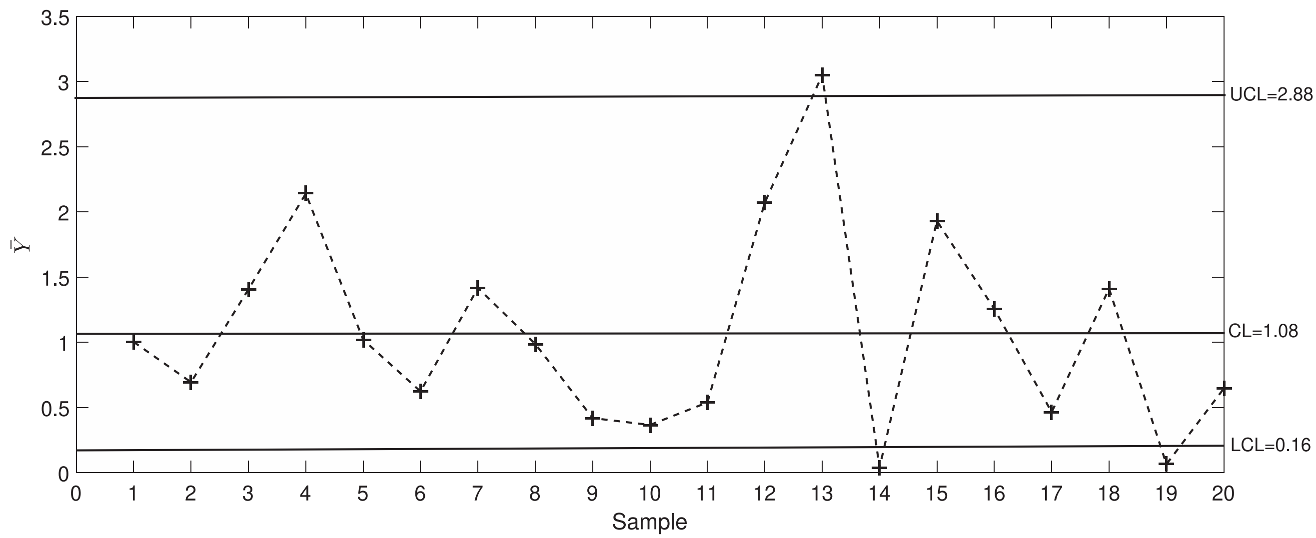

FIGURE 3 $\bar{Y}$ control chart for the numerical example

with the approach by Fernandes et al.[23] but has the advantage of being analytical and easily implemented even in spreadsheets, not requiring the use of Monte Carlo simulations. Unlike Fernandes et al.,[23] who considered only shifts in the scale parameter, in this paper, we also consider situations in which shifts occur on scale and shape parameters.

As a suggestion for future work, the expansion of the approach discussed in this article is suggested for dealing with cases in which the shape and scale parameters can change simultaneously. In this case, the control limits will remain the same as those presented here, although the analytical calculation of the ARL_1 will be challenging, since it seems quite complicated to obtain probabilities associated with the distribution of $\bar{Y}$ when the process is out of control, due to simultaneous changes in the parameters of scale and shape. ARL_1 calculations for situations with simultaneous changes in the parameters of scale and shape were discussed in this paper using a Monte Carlo simulation approach instead of an analytical approach. Another research possibility would be to include a supplementary run rule in order to improve the performance of the $\bar{Y}$ chart, similarly to what was developed in Ho et al.[43] and Khoo and Ariffin.[19]

AUTHOR CONTRIBUTIONS

The founders had no role in the study design, data collection and analysis, decision to publish, or preparation of the manuscript. All authors, Renata M. R. Vasconcelos, Roberto C. Quinino, Linda L. Ho, and Frederico R. B. Cruz contributed equally to the design and implementation of the research, to the analysis of the results, and to the final writing of the manuscript.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study. The data used to support the findings of this study are included within the article. The proposed algorithms can be encoded in the reader's favorite programming language. The R scripts can be obtained from the authors upon request.

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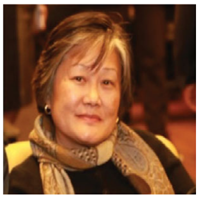

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